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String loop corrections to Kähler potentials in orientifolds

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Abstract: We determine one-loop string corrections to Kähler potentials in type-IIB orientifold compactifications with either $\mathcal{N} = 1$ or $\mathcal{N} = 2$ supersymmetry, including D-brane moduli, by evaluating string scattering amplitudes.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence, 1/N Expansion.

Contents

1. Introduction

Orientifolds represent an ideal laboratory to determine explicit loop corrections to low energy effective actions in string theory, due to their high degree of calculability.[1] In this

[1] See [1] and references therein for an introduction.

paper we determine string one-loop corrections to Kähler potentials in three different type-IIB orientifold models, one with $\mathcal{N}=2$ supersymmetry and two with $\mathcal{N}=1$. The results could find applications in various contexts from cosmology to particle phenomenology and we will exploit some of them in a companion paper [2].

Our main motivation for calculating string corrections to Kähler potentials is that they contribute to the scalar potential of the low energy effective action. This can have effects on the vacuum structure of the theory or on the dynamics of the scalar fields. The price one pays for the advantage of concrete calculability in orientifold models is the restriction to work at a special point in moduli space, the orbifold point. Nevertheless, our results give some important insights into the qualitative features arising from string corrections to Kähler potentials in general.

For instance, the loop corrections introduce new dependence of the Kähler potential on both the closed and open string moduli. Thus they can have immediate bearing on the issue of moduli stabilization. One interesting question in this context is whether perturbative corrections to the Kähler potential could lead to stabilization of the volume modulus without invoking non-perturbative effects such as gaugino condensation [3]. This possibility was recently also mentioned by [4]. To address this question one also has to take other sources for corrections to the Kähler potential than string loops into account, for instance α' corrections at string tree level. Some of these have been determined in [5]. We will see that the one-loop corrections to the Kähler potential in models with D3- and D7-branes are suppressed for large values of the volume compared to the leading tree level terms, but less suppressed than the α' corrections of [5]. Unless the string coupling is very small, they would thus generally be In [2] we only considered a superpotential induced by fluxes, i.e. no nonperturbative contributions. In that case it turns out that the leading correction to the *potential* (as opposed to the Kaehler potential) in a large volume expansion still comes from the α'-term. It is important to note that this is not in contradiction with the fact that the leading α' corrections in type-IIB string theory arise at order $\mathcal{O}(\alpha'^3)$. The corrections that we calculate only originate from world-sheets that are not present with only oriented closed strings (they come from D-branes and O-planes, i.e. annulus, Möbius and Klein bottle diagrams). We will come back to this question in our companion paper [2].

A second interesting application of our results lies in the field of brane inflation, initiated in [6 − 9] where an open string scalar plays the role of the inflaton field. Given the fact that the corrections depend on open string scalars, they open up new possibilities to find regions in moduli space where the mass of the inflaton takes a value that allows for slow roll inflation. The corrections may or may not alleviate the amount of fine tuning required to achieve that.

In practice we determine the corrections to the Kähler metrics of the scalar sigma-model by calculating 2-point functions of the relevant scalars (which include the open string scalars), following a similar calculation of Antoniadis, Bachas, Fabre, Partouche and Taylor, who considered a 2-point function of gravitons in [10].[2] As the metrics on the

[2]One way to do this is to relax momentum conservation [11 − 15]. We confirm the validity of this prescription in our case in appendix E by considering a 4-point function. In the case of [10], the result was

moduli spaces we consider are Kähler, it is convenient to use vertex operators directly for the Kähler variables, i.e. *Kähler structure adapted* vertex operators, cf. section 2.3. The Kähler variables for the orientifold model with $\mathcal{N} = 2$ supersymmetry were found in [10]. A crucial feature is that the definition of the Kähler variables for the Kähler moduli[3] involve a mixing between closed and open string scalars. A similar kind of mixing between geometric and non-geometric scalar fields in the definition of the appropriate Kähler variables is familiar from other circumstances, like from compactifications of the heterotic string (see for instance [17]) or from $\mathcal{N} = 2$ compactifications of type-II theories [18].

The moduli dependence of our result for the $\mathcal{N} = 2$ orientifold is in agreement with the one-loop Kähler metric given in [10] for the case of vanishing open string scalars, there inferred from the one-loop correction to the Einstein-Hilbert term.[4] As another check, we explicitly derive the one-loop correction to the prepotential from the corrected Kähler potential. This confirms that our result is consistent with supersymmetry.

Let us next summarize the content and organization of this paper. We consider the same three orientifold models as in [20], i.e. the $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$ model with $\mathcal{N} = 2$ supersymmetry (in section 2), the $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ model (in section 3) and the $\mathbb{T}^6/\mathbb{Z}_6'$ model (in section 4), both with $\mathcal{N} = 1$ supersymmetry. In section 2.4 we verify for the $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$ model that one can straightforwardly reproduce the tree level sigma-model metrics by calculating 2-point functions on the sphere and disk using the Kähler structure adapted vertex operators (doing so, we built on previous work on disk amplitudes, in particular [21 − 26]). We then continue by determining a particular 2-point function at one-loop that allows us to read off the one-loop Kähler potential. We perform several checks on the result. In section 2.6 we verify that the result is consistent with $\mathcal{N} = 2$ supersymmetry by determining the corresponding prepotential. A second check is performed in appendix C, where we calculate five other 2-point functions for the (vector multiplet) scalars of this model and show that the result is consistent with the proposed Kähler potential. Given the fact that the prepotential in $\mathcal{N} = 2$ theories only gets one-loop and non-perturbative corrections, we conclude that our result holds to all orders of perturbation theory in the $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$ case. The main result of section 2 is given in formula (2.77).

We then continue in sections 3 and 4 to generalize this result to the $\mathcal{N} = 1$ cases of the $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ and $\mathbb{T}^6/\mathbb{Z}_6'$ models. The main results in these sections can be found in formulas (3.30) and (4.3), respectively.

In section 5 we draw some conclusions and, in particular, translate our results to the T-dual picture with D3- and D7-branes.

Finally, we relegated some of the technical details to a series of appendices.

confirmed by a 3-point function of gravitons in [16].

[3] Notice the twofold use of the term "Kähler" here, because both the compactification manifold and the moduli space are Kähler.

[4] The case with D9-brane Wilson line moduli was also considered in [19]. Their integral representation for the Kähler metric (given in (2.4) of that paper) is less straightforward to use in applications, but it is consistent with the Kähler potential we find.

JHEP11(2005)030

2. The $\mathcal{N}=2$ orientifold $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$

We first study the type-IIB orientifold compactification with $\mathcal{N}=2$ supersymmetry on $\mathbb{T}^2\times$K3 at the orbifold point $\mathbb{T}^2 \times \mathbb{T}^4/\mathbb{Z}_2$ described in [27 − 29]. It is defined by world-sheet parity and the orbifold generator Θ of $\mathbb{Z}_2 = \{1, \Theta\}$ which acts on the coordinates of the $\mathbb{T}^4$ by reflection. More precisely, it acts on the complex coordinates along $\mathbb{T}^6 = \mathbb{T}^2_1 \times \mathbb{T}^2_2 \times \mathbb{T}^2_3$ by multiplication with $\exp(2\pi i\vec{v})$, where

$$\vec{v} = \left(0, \frac{1}{2}, -\frac{1}{2}\right). \tag{2.1}$$

The world-sheet parity operation Ω interchanges left- and right-moving fields on the closed string world-sheet, see [1] for a review. The model contains orientifold O9- and O5-planes, the former space-time filling, the latter localized at the fixed points of the orbifold generator. In the same way there are D9-branes and D5-branes wrapped on the $\mathbb{T}^2$ and point-like on the K3. We should stress that it is completely straightforward to translate the D9/D5 model into a model with D3- and D7-branes instead, by performing six T-dualities along all the internal directions; we will come back to this in the conclusions, section 5.

2.1 The classical lagrangian

The relevant aspects of the effective action that describes the low energy dynamics of the untwisted modes of this model have been discussed in [10]. The moduli of the K3 including all the blow-up modes of the orbifold singularities fall into $\mathcal{N}=2$ hypermultiplets and will not be important to us in the following. The scalars that arise from reducing the ten-dimensional fields along the $\mathbb{T}^2$ reside in vector multiplets, and their moduli space will be the subject of this section. Some of them arise from the closed string sector, but in addition there are the vector multiplets from the open string sectors of D9- and D5-branes. For our present purposes we will only consider the D9-brane scalars and set D5-brane scalars to zero, keeping the D5-brane gauge fields.

To be more specific, we focus on the complex scalars $\{S, S', U, A_i\}$ that are defined as

$$\begin{aligned} S &= \frac{1}{\sqrt{8\pi^2}}(C + ie^{-\Phi}\sqrt{G}\mathcal{V}_{\rm K3})\,, \\ U &= \frac{1}{G_{44}}(G_{45} + i\sqrt{G})\,, \qquad A^i = Ua^i_4 - a^i_5 \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} S' &= \frac{1}{\sqrt{8\pi^2}}(C_{45} + ie^{-\Phi}\sqrt{G}) + \frac{1}{8\pi}\sum_i N_i(U(a^i_4)^2 - a^i_4 a^i_5) \\ &= S'_0 + \frac{1}{8\pi}\sum_i N_i A_i \frac{A_i - \bar{A}_i}{U - \bar{U}}\,. \end{aligned} \tag{2.3}$$

We introduced the rescaled K3 volume in the string frame $\mathcal{V}_{\rm K3} = (4\pi^2\alpha')^{-2}{\rm vol(K3)}$, and used the string frame metric on $\mathbb{T}^2$,

$$G_{mn} = \frac{\sqrt{G}}{U_2}\begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix}. \tag{2.4}$$

Moreover, Φ is the ten-dimensional dilaton, C_{45} the internal component of the RR 2-form C_2 along the torus and $\partial_\mu C = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\nu C^{\rho\sigma}$. Finally, we denoted the internal components of the ten-dimensional abelian vectors of the D9-brane stack labelled by i as a^i_m (which are defined to be dimensionless, i.e. $\mathcal{F}^i_{\mu m} = \partial_\mu a^i_m/\sqrt{\alpha'}$), and N_i denotes the multiplicity of branes in the stack, i.e. the rank of the respective $\mathrm{U}(N_i)$ factor of the gauge group.[5] Note that the definition (2.3) of S' contains a correction involving the open string scalars, arising at disk level. We will review in a moment why this is a good Kähler variable in the presence of open string scalars.

The leading order interactions between the vector multiplets (coupled to gravity) can be derived from the dimensional reduction of ten-dimensional type-I supergravity [10]

$$\begin{aligned}\mathcal{S}_{\mathrm{SG}} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{10}}\left[e^{-2\Phi}\Big(R_{10}+4\partial^I\Phi\partial_I\Phi\Big)-\frac{1}{2}\left|dC_2-\frac{\kappa_{10}^2}{g_{10}^2}\omega_3\right|^2\right] \\ &= \int d^4x\sqrt{-g}\,\frac{1}{\kappa_4^2}\left[\frac{1}{2}R+\frac{\partial_\mu S\partial^\mu\bar{S}}{(S-\bar{S})^2}+\frac{\partial_\mu U\partial^\mu\bar{U}}{(U-\bar{U})^2}+\right. \\ &\qquad\left.+\frac{|\partial_\mu S'_0+\frac{1}{8\pi}\sum_i N_i(a^i_4\partial_\mu a^i_5-a^i_5\partial_\mu a^i_4)|^2}{(S'_0-\bar{S}'_0)^2}\right]+\cdots\,,\end{aligned}\tag{2.5}$$

together with the Born-Infeld action that produces the kinetic terms for the D9-brane gauge fields and scalars,

$$\begin{aligned}\mathcal{S}_{\mathrm{BI}} &= \mu_9\int d^{10}x\,e^{-\Phi}\Big[-\det(g_{IJ}+2\pi\alpha'(\mathcal{F}_{\mathrm{D9}})_{IJ})\Big]^{1/2} \\ &= \int d^4x\sqrt{-g}\left[\frac{1}{4\pi\kappa_4^2}\frac{\sum_i N_i|U\partial_\mu a^i_4-\partial_\mu a^i_5|^2}{(U-\bar{U})(S'_0-\bar{S}'_0)}-\frac{1}{4}\,\mathrm{Im}(S)\,\mathrm{tr}\,\mathcal{F}^2_{\mathrm{D9}}\right]+\cdots\,.\end{aligned}\tag{2.6}$$

The ellipsis stands for higher derivative terms, g_{IJ} denotes the ten-dimensional string frame metric and $g_{\mu\nu}$ the four-dimensional Einstein frame metric. All the non-abelian scalars are set to zero. The kinetic terms for D5-brane gauge fields are

$$\begin{aligned}\mathcal{S}_{\mathrm{BI}} &= \mu_5\int_{\mathbb{R}^{3,1}\times\mathbb{T}^2} d^6x\,e^{-\Phi}\Big[-\det(g_{IJ}+2\pi\alpha'(\mathcal{F}_{\mathrm{D5}})_{IJ})\Big]^{1/2} \\ &= \int d^4x\sqrt{-g}\left[-\frac{1}{4}\,\mathrm{Im}(S'_0)\,\mathrm{tr}\,\mathcal{F}^2_{\mathrm{D5}}\right]+\cdots\,.\end{aligned}\tag{2.7}$$

The volume of the background torus is taken to be $4\pi^2\alpha'\sqrt{G}$ and the constants are

$$\kappa_{10}^2=(4\pi^2\alpha')^3\kappa_4^2=\frac{1}{4\pi}(4\pi^2\alpha')^4\,,\qquad \mu_p=\frac{2\pi}{\sqrt{2}}(4\pi^2\alpha')^{-(p+1)/2}\,,\qquad \frac{\kappa_{10}^2}{g_{10}^2}=\frac{\alpha'}{2\sqrt{2}}\,.\tag{2.8}$$

The gauge kinetic terms are completed by the Chern-Simons action into the classical (i.e. disk level) holomorphic couplings

$$f^{(0)}_{\mathrm{D9}}=-iS\,,\qquad f^{(0)}_{\mathrm{D5}}=-iS'_0\,.\tag{2.9}$$

[5] For the open string scalars, we use the convention that i,j enumerate the different stacks of D9-branes, whereas m,n stand for the internal components along the torus, i.e. $m,n\in\{4,5\}$. Furthermore, we sometimes write the index i,j upstairs and sometimes downstairs, whichever is more convenient. We also do not follow the Einstein summation rule for i,j, always writing the summation explicitly.

Let us make some comments on the numerical factors appearing in (2.5) and (2.6). We have made all scalars dimensionless, hence the prefactors $\kappa_4^{-2} = (\pi\alpha')^{-1}$ in (2.5) and the first term of (2.6). Moreover, the S and S'_0 in (2.2) and (2.3) are defined such that the tree-level gauge kinetic functions are given by (2.9) without any numerical factors. This leaves one with the unconventional prefactor $(4\pi)^{-1}$ in (2.6) which matches precisely with the relative factor in the reduction of the Chern-Simons corrected kinetic term for the RR 2-form, i.e.

$$\frac{1}{\sqrt{8\pi^2}}\left[\partial_\mu C_{45} - \frac{\kappa_{10}^2}{g_{10}^2}(\omega_3)_{\mu 45}\right] = \frac{1}{\sqrt{8\pi^2}}\partial_\mu C_{45} + \frac{1}{2}\frac{1}{4\pi}\sum_i N_i(a_4^i\partial_\mu a_5^i - a_5^i\partial_\mu a_4^i)\,. \tag{2.10}$$

Putting the pieces together, the classical lagrangian that follows from the dimensional reduction of the leading order ten-dimensional supergravity action reads

$$\begin{aligned}\kappa_4^2\mathcal{L}_{4d} = {}& \frac{1}{2}R + \frac{\partial_\mu S\partial^\mu\bar{S}}{(S-\bar{S})^2} + \frac{\partial_\mu U\partial^\mu\bar{U}}{(U-\bar{U})^2} + \frac{|\partial_\mu S'_0 + \frac{1}{8\pi}\sum_i N_i(a_4^i\partial_\mu a_5^i - a_5^i\partial_\mu a_4^i)|^2}{(S'_0-\bar{S}'_0)^2} + \\ & + \frac{\sum_i N_i|U\partial_\mu a_4^i - \partial_\mu a_5^i|^2}{4\pi(U-\bar{U})(S'_0-\bar{S}'_0)} - \frac{1}{4}\kappa_4^2\,\mathrm{Im}(S)\,\mathrm{tr}\,\mathcal{F}_{\mathrm{D9}}^2 - \frac{1}{4}\kappa_4^2\,\mathrm{Im}(S'_0)\,\mathrm{tr}\,\mathcal{F}_{\mathrm{D5}}^2\,.\end{aligned} \tag{2.11}$$

It is important to observe that in the kinetic term of S'_0, there are terms of different dependence on the ten-dimensional dilaton, and with different numbers of traces over gauge group indices (i.e. factors of N_i). More precisely, the cross term

$$\sum_i N_i(a_4^i\partial_\mu a_5^i)\partial^\mu(\mathrm{Re}(S'_0)) + \cdots$$

has one trace and the dilaton dependence expected from open string tree level (disk diagrams), while the term with only open string scalars of the type

$$\sum_{i,j} N_iN_j(a_4^i\partial^\mu a_5^i)(a_4^j\partial_\mu a_5^j) + \cdots$$

has two traces and the dilaton dependence of an open string one-loop diagram. Due to these corrections to the kinetic term of S'_0 one has to use the modified field S' defined in (2.3) in order to make the Kähler property of the scalar sigma-model metric explicit. Thus the scalars $\{S, S', U, A_i\}$ are good Kähler variables to use at the classical level and their (classical) Kähler potential is

$$\begin{aligned}K^{(0)} &= -\ln(S-\bar{S}) - \ln\left[(S'_0-\bar{S}'_0)(U-\bar{U})\right] \\ &= -\ln(S-\bar{S}) - \ln\left[(S'-\bar{S}')(U-\bar{U}) - \frac{1}{8\pi}\sum_i N_i(A_i-\bar{A}_i)^2\right].\end{aligned} \tag{2.12}$$

As required by $\mathcal{N}=2$ supersymmetry, this can be expressed by a prepotential, i.e. by

$$\mathcal{F}^{(0)}(S,S',U,A_i) = S\left[S'U - \frac{1}{8\pi}\sum_i N_iA_i^2\right], \tag{2.13}$$

via the standard formula for the Kähler potential in special Kähler geometry

$$K = -\ln\left[2\mathcal{F} - 2\bar{\mathcal{F}} - \sum_\alpha (\phi^\alpha - \bar{\phi}^{\bar\alpha})(\mathcal{F}_\alpha + \bar{\mathcal{F}}_{\bar\alpha})\right], \tag{2.14}$$

ϕ^α running over all scalars $\{S, S', U, A_i\}$, and $\mathcal{F}_\alpha = \partial_{\phi^\alpha}\mathcal{F}$. Since some terms in (2.11) go as $(e^\Phi)^0$ in the string frame, which is characteristic of open string one-loop corrections, the moniker "classical" Kähler potential clearly has to be taken with a grain of salt in the type-I context (this was also emphasized in [20]). We will see, however, in section 2.4 that all the kinetic terms resulting from (2.12) can indeed be derived by calculating just tree diagrams (i.e. sphere and disk) if one uses appropriately defined Kähler structure adapted vertex operators, cf. section (2.3). We therefore continue to call (2.12) the tree-level (or classical) Kähler potential.

Finally, note that in $\mathcal{N} = 2$ supersymmetry the gauge couplings are also fixed by the prepotential and related to the Kähler metric of the matter fields in the same vector multiplet. The open string moduli are in general not the same as matter fields[6] but at points of enhanced gauge symmetry such as $A_i = 0$ they should be treated on the same footing. They are then expected to satisfy a relation [31]

$$2\pi i\, K_{A_i\bar{A}_i}\Big|_{A_i=0} = e^K \,\mathrm{Re}(f_{\mathrm{D9}_i})\Big|_{A_i=0}, \tag{2.15}$$

where the constant of proportionality has been fixed in a way that $K^{(0)}_{A_i\bar{A}_i}$ and $f^{(0)}_{\mathrm{D9}}$ respect the condition up to terms of order $\mathcal{O}(e^{2\Phi})$.

2.2 Classical Kähler metric

The tree level Kähler potential in $\mathbb{T}^4/\mathbb{Z}_2\times\mathbb{T}^2$ was already given in (2.12). In the following we will check and confirm it by performing explicit calculations of string scattering amplitudes on sphere and disk world-sheet topologies. This is a preliminary step before computing the one-loop corrections that will prove useful to develop some new techniques, in particular regarding choices of complex coordinates in the vertex operators for the relevant moduli fields. In our calculation we make use of results previously obtained in [21 – 26].

To make contact with the Kähler potential (2.12), we explicitly write out the components of the Kähler metric that result from it, immediately recasting them into the variables $\{S'_0, U, A_i\}$, which makes the dilaton dependence of the various terms obvious.[7] One finds

$$\begin{aligned}
K^{(0)}_{S'\bar{S}'} &= -\underbrace{\frac{1}{(S'_0 - \bar{S}'_0)^2}}_{\mathcal{O}(e^{2\Phi})},\\
K^{(0)}_{U\bar{U}} &= -\underbrace{\frac{1}{(U-\bar{U})^2}}_{\mathcal{O}(1)} - \frac{1}{4\pi}\underbrace{\frac{\sum_i N_i (A_i - \bar{A}_i)^2}{(U-\bar{U})^3 (S'_0 - \bar{S}'_0)}}_{\mathcal{O}(e^{\Phi})} - \frac{1}{64\pi^2}\underbrace{\frac{(\sum_i N_i (A_i - \bar{A}_i)^2)^2}{(U-\bar{U})^4 (S'_0 - \bar{S}'_0)^2}}_{\mathcal{O}(e^{2\Phi})},
\end{aligned}$$

[6] We adopt the convention of [30] who define matter fields as those that are charged under the gauge group and, thus, whose vacuum expectation values would break part of the gauge symmetry.

[7] We do not consider S, because it decouples at tree level, cf. (2.12).

$$
\begin{aligned}
K^{(0)}_{A_i\bar{A}_j} &= -\frac{1}{4\pi}\underbrace{\frac{N_i\delta_{ij}}{(U-\bar{U})(S'_0-\bar{S}'_0)}}_{\mathcal{O}(e^{\Phi})} - \frac{1}{16\pi^2}\underbrace{\frac{N_i(A_i-\bar{A}_i)N_j(A_j-\bar{A}_j)}{(U-\bar{U})^2(S'_0-\bar{S}'_0)^2}}_{\mathcal{O}(e^{2\Phi})},\\
K^{(0)}_{U\bar{S}'} &= -\frac{1}{8\pi}\underbrace{\frac{\sum_i N_i(A_i-\bar{A}_i)^2}{(U-\bar{U})^2(S'_0-\bar{S}'_0)^2}}_{\mathcal{O}(e^{2\Phi})},\\
K^{(0)}_{U\bar{A}_i} &= \frac{1}{4\pi}\underbrace{\frac{N_i(A_i-\bar{A}_i)}{(U-\bar{U})^2(S'_0-\bar{S}'_0)}}_{\mathcal{O}(e^{\Phi})} + \frac{1}{32\pi^2}\underbrace{\frac{N_i(A_i-\bar{A}_i)\sum_j N_j(A_j-\bar{A}_j)^2}{(U-\bar{U})^3(S'_0-\bar{S}'_0)^2}}_{\mathcal{O}(e^{2\Phi})},\\
K^{(0)}_{A_i\bar{S}'} &= \frac{1}{4\pi}\underbrace{\frac{N_i(A_i-\bar{A}_i)}{(U-\bar{U})(S'_0-\bar{S}'_0)^2}}_{\mathcal{O}(e^{2\Phi})}\,.
\end{aligned}
\tag{2.16}
$$

We have indicated to which order in the dilaton expansion the respective terms contribute. To read off the order in a perturbative expansion of the effective lagrangian, one has to take into account that for each derivative with respect to $\bar{S}'$ one also has a term $\partial_\mu \bar{S}'$ in the kinetic term that compensates one power of $e^{-\Phi}$. Another indicator for the order in perturbation theory of a given term is the number of traces (factors of N_i) that appear. As mentioned earlier, there are not only terms of sphere and disk order but also a number of terms that appear to originate from annulus diagrams, i.e. at order $e^{2\Phi}$ with two traces. Amidst this barrage of confusion, there is hope: we will show that the full expression can be obtained from a purely tree level (sphere + disk diagram) computation by using *Kähler structure adapted* vertex operators. They produce the additional factors of the string coupling and involve traces over gauge group indices.

In order to do so we will calculate a number of scattering amplitudes and show that they are consistent with the Kähler metric in (2.16). Concretely, we choose $K^{(0)}_{A_i\bar{A}_j}$ as an example and confirm the presence of both terms in this metric component. The other cases could be treated analogously. Since all tree-level 2-point functions vanish, we will have to calculate 3-point functions. The first step, though, is to determine the vertex operators.

2.3 Vertex operators for moduli fields

The correct vertex operators to represent the Kähler variables S', U, A can be determined by expressing the world-sheet action in terms of these fields and taking its variations. We write the relevant part of the bosonic world-sheet action

$$
\mathcal{S}_{\rm ws} = \frac{1}{2\pi\alpha'}\int_\Sigma d^2z\; G_{mn}\partial X^m\bar{\partial}X^n - \frac{i}{\sqrt{\alpha'}}\sum_B\int_{(\partial\Sigma)_B} d\theta\; q_B\, a_m^{s[B]}\dot{X}^m + \cdots\,, \tag{2.17}
$$

where the new label B enumerates the components of the boundaries (B takes values in the empty set for the sphere and Klein bottle, $B \in \{1\}$ for the disk and Möbius strip, and $B \in \{1,2\}$ for the annulus), $s[B]$ is the label for the stack of D-branes on which the boundary component B ends, and thus $\vec{a}^{\,s[B]} = (a_4^{s[B]}, a_5^{s[B]})$ denotes the (dimensionless) Wilson line modulus on the $s[B]$th stack of branes on which the Bth boundary ends.

Finally q_B takes values $(q_1, q_2) = (1, -1)$ for the annulus, distinguishing the two possible orientations (for the Möbius strip one would only have $q_1 = 1$). The metric G_{mn} on the two-torus was introduced in (2.4).

Let us explain our notation for the Wilson lines in more detail. The conventions are such that there are 32 D9-branes labelled by the Chan-Paton (CP) indices. This means that the Wilson lines of all the D9-branes are collectively written as 32×32 dimensional matrices W. All fields are then subject to projections onto invariant states under the world-sheet parity Ω and under Θ, which act on the W via the gamma-matrices γ_Ω and γ_Θ which we define later. They identify the upper and lower 16×16 blocks in W up to sign, and project out the off-diagonal blocks. In the end, the invariant Wilson lines are described by a single unconstrained 16×16 matrix which is just right to collect all the degrees of freedom of the adjoint representation of the maximal U(16) gauge group on the D9-branes. When some of the Wilson line scalars take non-trivial vacuum expectation values, the D9-branes are separated into stacks labelled by i and the gauge group is broken up into factors[8]

$$\mathrm{U}(16) \;\longrightarrow\; \prod_i \mathrm{U}(N_i)\,, \tag{2.18}$$

with

$$\sum_i N_i = 16\,. \tag{2.19}$$

We then break up all matrices into $(2N_i) \times (2N_i)$ blocks, denoted $\gamma_{\Omega i}$, $\gamma_{\Theta i}$ and W_i for instance. We are only interested in the scalars that behave like moduli in this situation, i.e. the abelian $\mathrm{U}(1)_i$ factors in $\mathrm{U}(N_i) = \mathrm{SU}(N_i) \times \mathrm{U}(1)_i$. These are described by matrices

$$W_i = \mathrm{diag}(\mathbf{1}_{N_i}, -\mathbf{1}_{N_i}) \oplus \mathbf{0}_{32-2N_i}\,, \tag{2.20}$$

and the matrix valued Wilson line vector along the two circles of the $\mathbb{T}^2$ is written as $\vec{a}^{\,i} W_i$, which has the property that it commutes with all matrices γ. The normalization is such that[9]

$$\mathrm{tr}(W_i W_j) = 2N_i \delta_{ij}\,. \tag{2.21}$$

Expressed in terms of the Kähler coordinates A^i and U, the $\vec{a}^i$ read

$$a_4^i = \frac{A^i - \bar{A}^i}{U - \bar{U}}\,, \qquad a_5^i = \frac{A^i \bar{U} - \bar{A}^i U}{U - \bar{U}}\,. \tag{2.22}$$

In order to proceed it is also useful to define the ordinary volume modulus of the $\mathbb{T}^2$ as $T = \sqrt{8\pi^2}(\mathrm{Re}(S_0') + ie^{\Phi}\,\mathrm{Im}(S_0'))$ such that $T_2 = \sqrt{G}$. For the moment, we keep G_{mn} and

[8]Similarly, one can also break the D5-brane gauge group U(16) into factors $\mathrm{U}(N_a)$ with $\sum_a N_a = 16$ but we are only dealing with the D9-brane Wilson line moduli explicitly.

[9]A different normalization would lead to a rescaling of the scalars A^i.

a_m^i as functions of T_2, and introduce S' later, by rewriting T_2 as a function of S', U, A^i and their conjugates. Then one has

$$\begin{aligned} G_{mn}\partial X^m \bar\partial X^n &= \partial\bar Z\bar\partial Z + \partial Z\bar\partial\bar Z \equiv f_1(T_2,U)\,, \\ \partial_{T_2} f_1(T_2,U) &= \frac{1}{T_2} f_1(T_2,U)\,, \\ \partial_U f_1(T_2,U) &= -\frac{2}{U-\bar U}\partial Z\bar\partial Z\,, \end{aligned} \tag{2.23}$$

and

$$\begin{aligned} a_m^{s[B]}\dot X^m &= \frac{\sqrt{2}}{(U-\bar U)^{1/2}(T-\bar T)^{1/2}}(A^{s[B]}\dot Z - \bar A^{s[B]}\dot{\bar Z}) \equiv f_2(U, A^{s[B]})\,, \\ \partial_{A^i} f_2(U, A^{s[B]}) &= \frac{\sqrt{2}\delta_{is[B]}}{(U-\bar U)^{1/2}(T-\bar T)^{1/2}}\dot Z\,, \\ \partial_U f_2(U, A^{s[B]}) &= -\frac{\sqrt{2}(A^{s[B]} - \bar A^{s[B]})}{(U-\bar U)^{3/2}(T-\bar T)^{1/2}}\dot Z\,, \end{aligned} \tag{2.24}$$

where, as in [25], we defined complexified coordinates

$$\begin{aligned} Z &= \sqrt{\frac{T_2}{2U_2}}(X^4 + \bar U X^5)\,, \qquad \bar Z = \sqrt{\frac{T_2}{2U_2}}(X^4 + U X^5)\,, \\ \Psi &= \sqrt{\frac{T_2}{2U_2}}(\psi^4 + \bar U\psi^5)\,, \qquad \bar\Psi = \sqrt{\frac{T_2}{2U_2}}(\psi^4 + U\psi^5)\,. \end{aligned} \tag{2.25}$$

To introduce S' we also have to express e^Φ in terms of T, U, A^i. According to [10], there is another scalar modulus given by the six-dimensional dilaton

$$e^{-2\Phi_6} = e^{-2\Phi}\mathcal{V}_{\mathrm{K3}} \tag{2.26}$$

which is part of a hypermultiplet. With this, we can express

$$T_2 = \sqrt{G} = -i\pi\sqrt{2}\left[\frac{(S_0' - \bar S_0')(S-\bar S)}{e^{-2\Phi_6}}\right]^{1/2}. \tag{2.27}$$

Inserting

$$S_0' - \bar S_0' = S' - \bar S' - \frac{1}{8\pi}\frac{\sum_i N_i (A_i - \bar A_i)^2}{U - \bar U} \tag{2.28}$$

and using

$$\left[\frac{S-\bar S}{(S_0' - \bar S_0')e^{-2\Phi_6}}\right]^{1/2} = e^\Phi\,, \tag{2.29}$$

one arrives at

$$\begin{aligned} \partial_U T_2 &= -i\frac{1}{8\sqrt{2}}\frac{\sum_i N_i(A_i - \bar A_i)^2}{(U-\bar U)^2}e^\Phi\,, \\ \partial_{A_i} T_2 &= i\frac{1}{4\sqrt{2}}\frac{N_i(A_i - \bar A_i)}{U - \bar U}e^\Phi\,, \\ \partial_{S_2'} T_2 &= \pi\sqrt{2}e^\Phi\,. \end{aligned} \tag{2.30}$$

Now we can compute the vertex operators from differentiating the full world-sheet action, obtaining

$$
\begin{aligned}
\frac{\delta\mathcal{S}_{\rm ws}}{\delta S_2'} &= \partial_{S_2'}T_2\,\partial_{T_2}\mathcal{S}_{\rm ws}\\
&= \frac{1}{2\pi\alpha'}\int_\Sigma d^2z\frac{i}{S_0'-\bar{S}_0'}(\partial\bar{Z}\bar{\partial}Z+\partial Z\bar{\partial}\bar{Z})\,,\\
\frac{\delta\mathcal{S}_{\rm ws}}{\delta U} &= \partial_U T_2\,\partial_{T_2}\mathcal{S}_{\rm ws}+\partial_U\mathcal{S}_{\rm ws}\\
&= \frac{1}{2\pi\alpha'}\int_\Sigma d^2z\left[\frac{1}{16\pi}\frac{\sum_i N_i(A_i-\bar{A}_i)^2}{(U-\bar{U})^2(S_0'-\bar{S}_0')}(\partial\bar{Z}\bar{\partial}Z+\partial Z\bar{\partial}\bar{Z})-\frac{2}{U-\bar{U}}\partial Z\bar{\partial}Z\right]+\\
&\quad+\frac{i}{\sqrt{\alpha'}}\sum_B\int_{(\partial\Sigma)_B}d\theta\;q_B\frac{\sqrt{2}(A_{s[B]}-\bar{A}_{s[B]})}{(U-\bar{U})^{3/2}(T-\bar{T})^{1/2}}\dot{Z}\,,\\
\frac{\delta\mathcal{S}_{\rm ws}}{\delta A_j} &= \partial_{A_j}T_2\,\partial_{T_2}\mathcal{S}_{\rm ws}+\partial_{A_j}\mathcal{S}_{\rm ws}\\
&= \frac{1}{2\pi\alpha'}\int_\Sigma d^2z\left[-\frac{1}{8\pi}\frac{N_j(A_j-\bar{A}_j)}{(U-\bar{U})(S_0'-\bar{S}_0')}(\partial\bar{Z}\bar{\partial}Z+\partial Z\bar{\partial}\bar{Z})\right]-\\
&\quad-\frac{i}{\sqrt{\alpha'}}\sum_B\int_{(\partial\Sigma)_B}d\theta\;q_B\frac{\sqrt{2}\delta_{js[B]}}{(U-\bar{U})^{1/2}(T-\bar{T})^{1/2}}\dot{Z}\,.
\end{aligned}
\tag{2.31}
$$

Let us define a set of "building block" vertex operators, in the 0 picture

$$
\begin{aligned}
V^{(0,0)}_{Z\bar{Z}} &= -\frac{2}{\alpha'}\int_\Sigma d^2z\,\left[i\partial\bar{Z}+\frac{1}{2}\alpha'(p\cdot\psi)\bar{\Psi}\right]\left[i\bar{\partial}Z+\frac{1}{2}\alpha'(p\cdot\tilde{\psi})\tilde{\Psi}\right]e^{ipX}-\\
&\quad-\frac{2}{\alpha'}\int_\Sigma d^2z\,\left[i\partial Z+\frac{1}{2}\alpha'(p\cdot\psi)\Psi\right]\left[i\bar{\partial}\bar{Z}+\frac{1}{2}\alpha'(p\cdot\tilde{\psi})\bar{\tilde{\Psi}}\right]e^{ipX}\,,\\
V^{(0,0)}_{ZZ} &= -\frac{2}{\alpha'}\int_\Sigma d^2z\,\left[i\partial Z+\frac{1}{2}\alpha'(p\cdot\psi)\Psi\right]\left[i\bar{\partial}Z+\frac{1}{2}\alpha'(p\cdot\tilde{\psi})\tilde{\Psi}\right]e^{ipX}\,,\\
V^{(0,0)}_{\bar{Z}\bar{Z}} &= -\frac{2}{\alpha'}\int_\Sigma d^2z\,\left[i\partial\bar{Z}+\frac{1}{2}\alpha'(p\cdot\psi)\bar{\Psi}\right]\left[i\bar{\partial}\bar{Z}+\frac{1}{2}\alpha'(p\cdot\tilde{\psi})\bar{\tilde{\Psi}}\right]e^{ipX}\,,\\
V^{(0)B}_{Z} &= \frac{1}{\sqrt{2\alpha'}}\lambda_{s[B]}\int_{(\partial\Sigma)_B}d\theta\;[i\dot{Z}+2\alpha'(p\cdot\psi)\Psi]e^{ipX}\,,\\
V^{(0)B}_{\bar{Z}} &= \frac{1}{\sqrt{2\alpha'}}\lambda^\dagger_{s[B]}\int_{(\partial\Sigma)_B}d\theta\;[i\dot{\bar{Z}}+2\alpha'(p\cdot\psi)\bar{\Psi}]e^{ipX}\,,
\end{aligned}
\tag{2.32}
$$

and in the (-1) picture

$$
\begin{aligned}
V^{(-1,-1)}_{Z\bar{Z}} &= \int_\Sigma d^2z\;e^{-\phi-\tilde{\phi}}\bar{\Psi}\tilde{\Psi}e^{ipX}+\int_\Sigma d^2z\;e^{-\phi-\tilde{\phi}}\Psi\bar{\tilde{\Psi}}e^{ipX}\,,\\
V^{(-1,-1)}_{ZZ} &= \int_\Sigma d^2z\;e^{-\phi-\tilde{\phi}}\Psi\tilde{\Psi}e^{ipX}\,,\\
V^{(-1,-1)}_{\bar{Z}\bar{Z}} &= \int_\Sigma d^2z\;e^{-\phi-\tilde{\phi}}\bar{\Psi}\bar{\tilde{\Psi}}e^{ipX}\,,\\
V^{(-1)B}_{Z} &= \lambda_{s[B]}\int_{(\partial\Sigma)_B}d\theta\;e^{-\phi}\Psi e^{ipX}\,,
\end{aligned}
$$

$$V_{\bar{Z}}^{(-1)B} = \lambda_{s[B]}^{\dagger} \int_{(\partial\Sigma)_B} d\theta \; e^{-\phi}\bar{\Psi} e^{ipX} \,. \tag{2.33}$$

We then define the *Kähler structure adapted* vertex operators corresponding to the Kähler coordinates

$$\begin{aligned}
V_{S_2'}^{(l,m)} &= g_c \alpha'^{-2} \frac{i}{S_0' - \bar{S}_0'} V_{Z\bar{Z}}^{(l,m)} \,, \\
V_U^{(l,m;n)} &= g_c \alpha'^{-2} \bigg(\frac{1}{16\pi} \frac{\sum_i N_i (A_i - \bar{A}_i)^2}{(U-\bar{U})^2 (S_0' - \bar{S}_0')} V_{Z\bar{Z}}^{(l,m)} - \frac{2}{U-\bar{U}} V_{ZZ}^{(l,m)} + \\
&\qquad + \sum_B \frac{4\pi (A_{s[B]} - \bar{A}_{s[B]})}{(U-\bar{U})^{3/2} (T-\bar{T})^{1/2}} V_Z^{(n)B} \bigg) \,, \\
V_{\bar{U}}^{(l,m;n)} &= g_c \alpha'^{-2} \bigg(-\frac{1}{16\pi} \frac{\sum_i N_i (A_i - \bar{A}_i)^2}{(U-\bar{U})^2 (S_0' - \bar{S}_0')} V_{Z\bar{Z}}^{(l,m)} + \frac{2}{U-\bar{U}} V_{\bar{Z}\bar{Z}}^{(l,m)} - \\
&\qquad - \sum_B \frac{4\pi (A_{s[B]} - \bar{A}_{s[B]})}{(U-\bar{U})^{3/2} (T-\bar{T})^{1/2}} V_{\bar{Z}}^{(n)B} \bigg) \,, \\
V_{A_j}^{(l,m;n)} &= g_o \alpha'^{-2} \bigg(-\frac{1}{8\pi^2} \frac{N_j (A_j - \bar{A}_j)}{(U-\bar{U})(S_0' - \bar{S}_0')} V_{Z\bar{Z}}^{(l,m)} - \\
&\qquad - \sum_B \frac{4\delta_{js[B]}}{(U-\bar{U})^{1/2} (T-\bar{T})^{1/2}} V_Z^{(n)B} \bigg) \,, \\
V_{\bar{A}_j}^{(l,m;n)} &= g_o \alpha'^{-2} \bigg(\frac{1}{8\pi^2} \frac{N_j (A_j - \bar{A}_j)}{(U-\bar{U})(S_0' - \bar{S}_0')} V_{Z\bar{Z}}^{(l,m)} + \\
&\qquad + \sum_B \frac{4\delta_{js[B]}}{(U-\bar{U})^{1/2} (T-\bar{T})^{1/2}} V_{\bar{Z}}^{(n)B} \bigg) \,,
\end{aligned} \tag{2.34}$$

where the explicit factors of α' are chosen such that the vertex operators are dimensionless and we allowed for different superghost pictures for the closed and open string parts of the vertex operators. In practice we will always use the vertex operators for $l = m = n \in \{0, -1\}$. For S' we only write down the vertex operator of the imaginary part S_2' because the real part is given by an RR-field and does not appear in the world sheet action (2.17). Note, moreover, that the charges q_B have been absorbed in the definition of the Chan-Paton matrices $\lambda_{s[B]}$ appearing in the building block vertex operators (2.32) and (2.33).

A striking feature of the expressions (2.34) is the fact that the vertex operators for the Kähler coordinates are combinations of open and closed string vertex operators (except for S_2') and thus contribute all in string diagrams with or without boundaries. For instance, the vertex operator for an open string scalar A_i can appear not only in a disk diagram but also in a sphere diagram at tree-level via the term proportional to $V_{Z\bar{Z}}^{(l,m)}$. Furthermore, the coefficients involve powers of the dilaton and thus the naive counting of the order (in an expansion in the dilaton) at which a certain diagram contributes is no longer valid when the Kähler structure adapted vertex operators are inserted. The reason for this is, of course, the redefinition of S' at disk level (2.3). This feature of the vertex operators (2.34) allows us to derive even those terms in (2.16) from tree level diagrams that usually (i.e. when using unadapted vertex operators) would arise only at one-loop level.

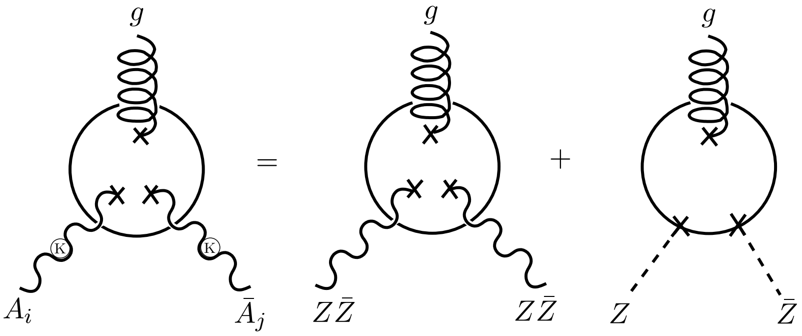

Figure 1: Computing a disk 3-point function using Kähler structure adapted vertex operators (denoted by Ⓚ). The adapted vertex operators contain both open and closed vertex operators.

2.4 Tree level diagrams

Now we are in a position to calculate 3-point functions on the sphere and disk. As an example we consider the case of 3-point functions with one graviton and two open string scalars, from which one can read off the sigma-model metric $K^{(0)}_{A_i\bar{A}_j}$ of (2.16). We thus obtain on the sphere

$$\begin{aligned}
&\langle V_g^{(0,0)} V_{A_i}^{(-1,-1;-1)} V_{\bar{A}_j}^{(-1,-1;-1)} \rangle_{\text{sphere}} = \\
&\quad = -\frac{g_o^2\alpha'^{-4}}{64\pi^4} \frac{N_i(A_i-\bar{A}_i)N_j(A_j-\bar{A}_j)}{(U-\bar{U})^2(S_0'-\bar{S}_0')^2} \langle V_g^{(0,0)} V_{Z\bar{Z}}^{(-1,-1)} V_{Z\bar{Z}}^{(-1,-1)} \rangle_{\text{sphere}}\,, \qquad (2.35)
\end{aligned}$$

whereas on the disk we have to calculate[10] (see figure 1)

$$\begin{aligned}
&\langle V_g^{(-1,-1)} V_{A_i}^{(0,0;0)} V_{\bar{A}_j}^{(0,0;0)} \rangle_{\text{disk}} = \\
&\quad = -\frac{g_o^2\alpha'^{-4}}{64\pi^4} \frac{N_i(A_i-\bar{A}_i)N_j(A_j-\bar{A}_j)}{(U-\bar{U})^2(S_0'-\bar{S}_0')^2} \langle V_g^{(-1,-1)} V_{Z\bar{Z}}^{(0,0)} V_{Z\bar{Z}}^{(0,0)} \rangle_{\text{disk}} - \\
&\quad\quad -\frac{16 g_o^2\alpha'^{-4}\delta_{ij}}{(U-\bar{U})(T-\bar{T})} \langle V_g^{(-1,-1)} V_Z^{(0)} V_{\bar{Z}}^{(0)} \rangle_{\text{disk}}\,. \qquad (2.36)
\end{aligned}$$

The graviton vertex operators are given by

$$\begin{aligned}
V_g^{(0,0)} &= -\frac{2g_c}{\alpha'}\epsilon_{\mu\nu}\int_\Sigma d^2z \left[i\partial X^\mu + \frac{1}{2}\alpha'(p\cdot\psi)\psi^\mu\right]\left[i\bar{\partial}X^\nu + \frac{1}{2}\alpha'(p\cdot\tilde{\psi})\tilde{\psi}^\nu\right] e^{ipX}\,, \\
V_g^{(-1,-1)} &= g_c\epsilon_{\mu\nu}\int_\Sigma d^2z\; e^{-\phi-\tilde{\phi}}\psi^\mu\tilde{\psi}^\nu e^{ipX}\,, \qquad (2.37)
\end{aligned}$$

for a symmetric, transverse and traceless polarization tensor $\epsilon_{\mu\nu}$. Moreover, we use the same world-sheet correlators as in [32], i.e.

$$\begin{aligned}
\langle X^M(z_1)X^N(z_2)\rangle &= -\frac{\alpha'}{2}g^{MN}\ln|z_{12}|^2\,, \\
\langle \psi^M(z_1)\psi^N(z_2)\rangle &= g^{MN}z_{12}^{-1}\,,
\end{aligned}$$

[10]As there is just one boundary on the disk we omit the index B in this section.

JHEP11(2005)030

$$
\begin{aligned}
\langle \tilde{\psi}^M(\bar{z}_1)\tilde{\psi}^N(\bar{z}_2)\rangle &= g^{MN}\bar{z}_{12}^{-1}\,, \\
\langle e^{-\phi}(z_1)e^{-\phi}(z_2)\rangle &= z_{12}^{-1}\,, \\
\langle e^{-\tilde{\phi}}(\bar{z}_1)e^{-\tilde{\phi}}(\bar{z}_2)\rangle &= \bar{z}_{12}^{-1}\,,
\end{aligned} \tag{2.38}
$$

where M, N can stand either for the external coordinates or the torus directions and $z_{12} = z_1 - z_2$. On the disk there are further contributions coming from

$$
\begin{aligned}
\langle \bar{\partial} X^M(\bar{z}_1)\partial X^N(z_2)\rangle_{\text{disk}} &= -\frac{\alpha'}{2} g^{MN}(\bar{z}_1 - z_2)^{-2}\,, \\
\langle \tilde{\psi}^M(\bar{z}_1)\psi^N(z_2)\rangle_{\text{disk}} &= g^{MN}(\bar{z}_1 - z_2)^{-1}\,.
\end{aligned} \tag{2.39}
$$

Notice also that the correlators for the complexified variables (2.25) follow as [25]

$$
\begin{aligned}
\langle \partial Z(z_1)\partial \bar{Z}(z_2)\rangle &= -\frac{\alpha'}{2} z_{12}^{-2}\,, \\
\langle \bar{\partial} Z(\bar{z}_1)\bar{\partial}\bar{Z}(\bar{z}_2)\rangle &= -\frac{\alpha'}{2}\bar{z}_{12}^{-2}\,, \\
\langle \bar{\partial} Z(\bar{z}_1)\partial \bar{Z}(z_2)\rangle &= \langle \bar{\partial}\bar{Z}(\bar{z}_1)\partial Z(z_2)\rangle = -\frac{\alpha'}{2}(\bar{z}_1 - z_2)^{-2}\,,
\end{aligned} \tag{2.40}
$$

and

$$
\begin{aligned}
\langle \Psi(z_1)\bar{\Psi}(z_2)\rangle &= z_{12}^{-1}\,, \\
\langle \tilde{\Psi}(\bar{z}_1)\bar{\tilde{\Psi}}(\bar{z}_2)\rangle &= \bar{z}_{12}^{-1}\,, \\
\langle \tilde{\Psi}(\bar{z}_1)\bar{\Psi}(z_2)\rangle &= \langle \bar{\tilde{\Psi}}(\bar{z}_1)\Psi(z_2)\rangle = (\bar{z}_1 - z_2)^{-1}\,.
\end{aligned} \tag{2.41}
$$

All other correlators, i.e. those for purely holomorphic or anti-holomorphic fields, vanish. Now we are ready to calculate (2.35) and (2.36). Fixing the positions of the three vertex operators in (2.35) at $0, 1$ and ∞, respectively, and including the corresponding ghost factor leads to (cf. [32])

$$
\langle V_g^{(0,0)} V_{A_i}^{(-1,-1;-1)} V_{\bar{A}_j}^{(-1,-1;-1)}\rangle_{\text{sphere}} \sim \frac{g_c g_o^2 \alpha'^{-3} N_i(A_i - \bar{A}_i)N_j(A_j - \bar{A}_j)}{(U - \bar{U})^2(S_0' - \bar{S}_0')^2}\epsilon_{\mu\nu}\, p_{23}^{\mu}\, p_{23}^{\nu}\,, \tag{2.42}
$$

where $p_{23}^{\mu} = p_2^{\mu} - p_3^{\mu}$. This result confirms the presence of the second term of $K^{(0)}_{A_i\bar{A}_j}$ of (2.16). We do not bother about determining the overall factor here, because the main purpose of this exercise is to show that the modification of the vertex operators is essential for deriving the terms with the right moduli dependence in the sigma-model metrics (2.16).

The first term of (2.36) actually does not contribute to the sigma-model metric. However, in order to see this, it is much easier to consider the 2-point function

$$
\langle V_{A_i}^{(-1,-1;-1)} V_{\bar{A}_j}^{(0,0;0)}\rangle_{\text{disk}} = -\frac{g_o^2\alpha'^{-4}}{64\pi^2}\frac{N_i(A_i - \bar{A}_i)N_j(A_j - \bar{A}_j)}{(U - \bar{U})^2(S_0' - \bar{S}_0')^2}\langle V_{Z\bar{Z}}^{(-1,-1)} V_{Z\bar{Z}}^{(0,0)}\rangle_{\text{disk}}\,. \tag{2.43}
$$

This does not vanish automatically due to the infinite volume of an unfixed conformal Killing group, as in the case of the closed string 2-point function on the sphere or the open string 2-point function on the disk. However, it would vanish on-shell. To deal with this, one can attempt to relax momentum conservation. Concretely, setting $\delta = p_1 \cdot p_2 \neq 0$ and expanding the result to linear order in δ, one can try to read off the metric as the coefficient of δ. A similar procedure was used in [11, 13] to investigate the generation of one-loop mass terms of scalar fields in heterotic and type-I theories, in [14] to compute anomalous dimensions and Kähler metric renormalization for matter fields in orientifolds, and in [15] to calculate the mass terms for anomalous U(1) gauge fields in type-I compactifications.[11] The relevant amplitude was already calculated in appendix A.2 of [25] with the result that there are no contributions to linear order in δ.[12]

The second term of (2.36) does, however, contribute to the sigma-model metric of the open string scalars. Again, we do not keep track of the exact prefactors. The result can be easily extracted from section 5 of [22] (in particular from their formulas (29) and (30)). Using that all the momenta are external and the polarization of the open string scalars is along the untwisted internal 2-torus, all contractions between momenta and scalar polarization can be discarded. The same is true for contractions between polarizations of the scalars and the graviton (which has external indices like the momenta). Finally, using momentum conservation (which does not need to be relaxed for this 3-point function) and tracelessness and transversality of the graviton polarization tensor, it is easy to see that the only contribution in our case is of the form

$$\langle V_g^{(-1,-1)} V_{A_i}^{(0,0;0)} V_{\bar{A}_j}^{(0,0;0)} \rangle_{\rm disk} \sim \frac{g_c g_o^2 \alpha'^{-3} N_i \delta_{ij}}{(U-\bar{U})(T-\bar{T})} \epsilon_{\mu\nu}\, p_{23}^{\mu}\, p_{23}^{\nu}\,, \tag{2.44}$$

where the factor $N_i \delta_{ij}$ comes from the trace over CP labels. Being a disk diagram, the above correlator gets an additional factor e^{Φ} relative to the sphere after performing the Weyl rescaling to Einstein frame, which promotes $T - \bar{T}$ to $S_0' - \bar{S}_0'$. This confirms the presence of the first term of $K^{(0)}_{A_i \bar{A}_j}$ of (2.16).

2.5 One-loop diagrams

After establishing the correct form for the vertex operators and clarifying the status of the tree-level action we now come to the main purpose of this paper: the calculation of one-loop corrections from Klein bottle, annulus and Möbius strip diagrams. We have computed all the relevant 2-point functions which allows us to read off the correction terms for all the components of the Kähler metric whose classical terms are given in (2.16). To determine the Kähler potential it turns out to be sufficient to compute a single such component and integrate. For the sake of transparency, we have chosen to present the simplest 2-point function in this section, and collected all the other amplitudes in appendix C. There we

[11] A similar method was also used in [33] and in [12] for calculations of 3-point functions at one-loop in the heterotic string. As pointed out in [12], the reason why momentum-conservation relaxation works particularly well for 2-point functions is that there is little ambiguity in how to go off-shell. For further discussion, see also section 13 of [34].

[12] One more observation is worth making. In the case of Dp-branes with $p < 9$, the amplitude contains a term proportional to s/t, where s is the square of the momenta parallel to the brane, $s = 2(p_1^{||})^2 = 2(p_2^{||})^2$, and $t = p_1 \cdot p_2$. This t-channel pole arises when the two vertex operators come together, and the coefficient of s/t must be proportional to the sigma-model metric at sphere level, cf. figure 2 in [22]. Comparing the moduli dependence in (2.35) and (2.43) we see that this proportionality also holds in our case.

JHEP11(2005)030

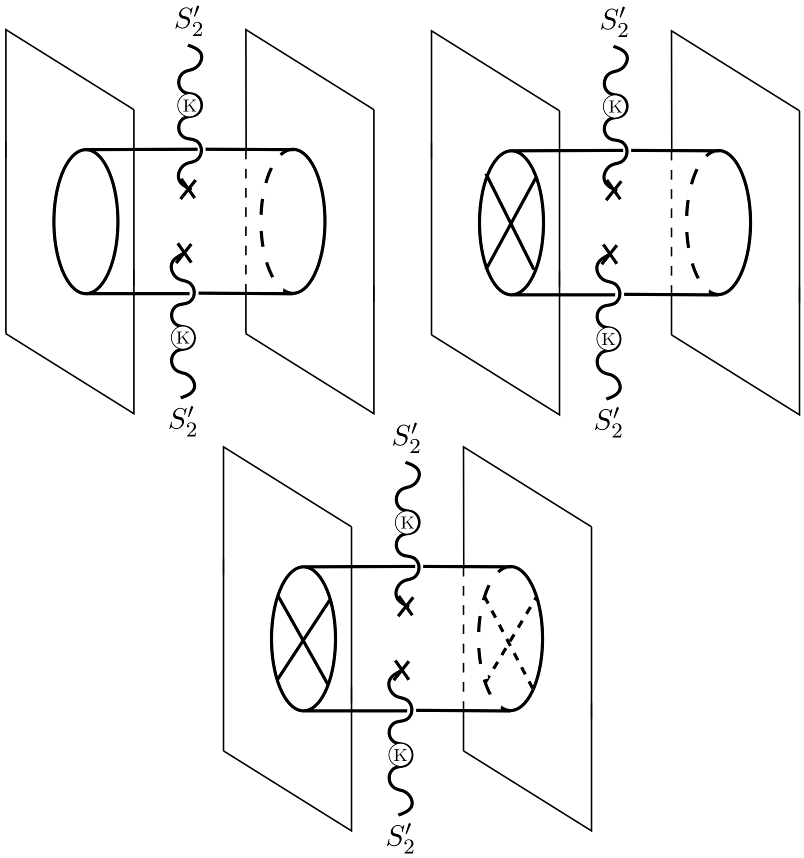

Figure 2: $\langle V_{S_2'} V_{S_2'} \rangle_\sigma$ for $\sigma = \mathcal{A}, \mathcal{M}, \mathcal{K}$, with Kähler adapted vertex operators.

will show that the other correlation functions are consistent with the Kähler potential derived from the one presented in this section.

In this section we calculate the 2-point function for S_2' on all one-loop surfaces: the torus, Klein bottle, annulus and Möbius strip. This is sufficient to determine the Kähler potential on the moduli space of vector moduli, as we will see in section 2.6. The calculation again necessitates the use of the off-shell prescription introduced in the last section for the 2-point function on the disk (2.43). More precisely, now we want to compute

$$\langle V_{S_2'} V_{S_2'} \rangle = -g_c^2 \alpha'^{-4} \frac{1}{(S_0' - \bar{S}_0')^2} \sum_\sigma \langle V_{Z\bar{Z}}^{(0,0)} V_{Z\bar{Z}}^{(0,0)} \rangle_\sigma \,, \tag{2.45}$$

where the index σ is used to label different types of amplitudes. This 2-point function is depicted in figure 2. We enumerate the contributions of the various diagrams symbolically in the form

$$\mathcal{K} + \mathcal{A} + \mathcal{M} = \sum_{k=0,1} \left[\mathcal{K}_1^{(k)} + \mathcal{K}_\Theta^{(k)} + \mathcal{A}_{99}^{(k)} + \mathcal{A}_{55}^{(k)} + \mathcal{A}_{95}^{(k)} + \mathcal{A}_{59}^{(k)} + \mathcal{M}_9^{(k)} + \mathcal{M}_5^{(k)} \right], \tag{2.46}$$

where warn the reader that we do not mean partition functions, but the above correlator $\langle V_{Z\bar{Z}}^{(0,0)} V_{Z\bar{Z}}^{(0,0)} \rangle_\sigma$ of (2.45) evaluated on these world-sheets. There is no contribution from the torus [10]. The upper index k stands for the power of Θ inserted in the trace (coming from the orbifold projector $\mathcal{P} = \frac{1}{2}(1+\Theta)$). The Klein bottle contains a sum over all 16 twisted sectors at the fixed points of Θ on the $\mathbb{T}^4$. When we break up the D9-branes (and potentially D5-branes) into stacks, we use the notation

$$\begin{aligned} \mathcal{A}_{99}^{(k)} &= \sum_{i,j} \mathcal{A}_{ij}^{(k)}\,, \qquad \mathcal{A}_{95}^{(k)} = \sum_{i,a} \mathcal{A}_{ia}^{(k)}\,, \qquad \mathcal{A}_{55}^{(k)} = \sum_{a,b} \mathcal{A}_{ab}^{(k)}\,, \\ \mathcal{M}_{9}^{(k)} &= \sum_{i} \mathcal{M}_{i}^{(k)}\,, \qquad \mathcal{M}_{5}^{(k)} = \sum_{a} \mathcal{M}_{a}^{(k)}\,. \end{aligned} \tag{2.47}$$

In this notation σ takes values $\sigma \in \{(ij),(ab),(ia),(ai)\}$ for annulus diagrams, $\sigma \in \{(i),(a)\}$ for Möbius diagrams, and $\sigma \in \{(\Theta),(1)\}$ for the twisted and untwisted sectors of the Klein bottle, respectively. Each diagram can have a different (and characteristic) dependence on the Wilson lines. We therefore also introduce a symbolic notation for the Wilson lines, writing $\vec{\mathbf{A}}_\sigma$ for the matrix valued Wilson lines, which now also become tensor valued in the case of annulus diagrams, where they can appear at both of the two ends individually. We will provide a list of these expressions later in table 1.

The correlator in (2.45) is now expanded in powers of $\delta = p_1 \cdot p_2$, and only the linear term is relevant for the Kähler metric. The sum over spin structures causes the amplitudes to vanish unless we contract at least four of the world sheet fermions appearing in $V_{Z\bar{Z}}^{(0,0)}$. This already gives the momentum dependence we want and we set $p=0$ elsewhere, as in [10].[13] Doing this we find that (2.45) can be expressed in the form

$$\begin{aligned} \langle V_{Z\bar{Z}}^{(0,0)} V_{Z\bar{Z}}^{(0,0)} \rangle_\sigma &= -V_4 \frac{(p_1\cdot p_2)\sqrt{G}}{8(4\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^4} \int_{\mathcal{F}_\sigma} d^2\nu_1 d^2\nu_2 \times \\ & \times \sum_{k=0,1} \sum_{\vec{n}=(n,m)^T} \mathrm{tr} \Bigg[e^{-\pi \vec{n}^T G \vec{n} t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma \cdot \vec{n}} \sum_{\substack{\alpha\beta \\ \text{even}}} \frac{\vartheta\Big[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\Big](0,\tau)}{\eta^3(\tau)} \gamma_{\sigma,k} \mathcal{Z}^{\text{int}}_{\sigma,k}\Big[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\Big] \times \\ & \times \Big[\langle \bar\partial Z(\bar\nu_1) \bar\partial \bar Z(\bar\nu_2)\rangle_\sigma \langle \bar\Psi(\nu_1)\Psi(\nu_2)\rangle_\sigma^{\alpha,\beta} \times \\ & \times \langle \psi(\nu_1)\psi(\nu_2)\rangle_\sigma^{\alpha,\beta} + \langle \bar\partial Z(\bar\nu_1) \partial \bar Z(\nu_2)\rangle_\sigma \times \\ & \langle \bar\Psi(\nu_1)\tilde{\bar\Psi}(\bar\nu_2)\rangle_\sigma^{\alpha,\beta} \langle \psi(\nu_1)\tilde\psi(\bar\nu_2)\rangle_\sigma^{\alpha,\beta} + \text{c.c.}\Big]\Bigg] + \mathcal{O}(\delta^2)\,. \end{aligned} \tag{2.48}$$

Many comments are in order here. V_4 denotes the regulated volume of the four-dimensional spacetime. We changed variables on the world-sheet and took all vertex operators (2.32) to depend on the coordinate ν, which is related to z by $z = e^{-i\nu}$. This choice coincides with the convention of [32] but differs from the one of [10] by a factor of 2π in the exponent; cf. appendix A for more details on our world-sheet conventions, in particular figure 3. The sum over bosonic zero modes has been made explicit, since there is also an implicit dependence on m, n in the bosonic correlators: this arises from the classical piece in the split into zero

[13] In general, a shortcut like this can be invalidated by poles from the integration over vertex operator positions. We check the validity of this procedure in one example, by calculating a 4-point function in appendix E.

JHEP11(2005)030

modes and fluctuations. That is, $Z(\nu) = Z_{\rm class}(\nu) + Z_{\rm qu}(\nu)$, where the classical part is given by

$$Z_{\rm class} = \sqrt{\alpha'}\sqrt{\frac{T_2}{2U_2}}\frac{(n+m\bar{U})}{\tau_2}\,{\rm Im}(\nu)\,c_\sigma\,, \qquad c_\sigma = \begin{cases} 1 & \text{for } \mathcal{A}\,,\mathcal{M} \\ 2 & \text{for } \mathcal{K} \end{cases}\,. \tag{2.49}$$

These zero modes have the right periodicity under ${\rm Im}(\nu) \to {\rm Im}(\nu) + 2\pi\tau_2$ (for $\mathcal{A}, \mathcal{M}$) or ${\rm Im}(\nu) \to {\rm Im}(\nu) + \pi\tau_2$ (for $\mathcal{K}$), i.e. $X^4 \to X^4 + 2\pi n\sqrt{\alpha'}$ and $X^5 \to X^5 + 2\pi m\sqrt{\alpha'}$. The zero modes do not have any analogue at tree level; one needs a non-trivial 1-cycle on the world-sheet. They do play an important role in calculating the moduli dependence of one-loop corrections to the gauge couplings in the heterotic string performed in [33] and reviewed in, for instance, [34].

The internal partition function is abbreviated $\mathcal{Z}^{\rm int}_{\sigma,k}$ for the diagram σ with insertion Θ^k and carries a label α, β for the spin structure.[14] For the annulus and Möbius strip diagrams it is [28, 35, 36]

$$\mathcal{Z}^{\rm int}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \eta_{\alpha\beta}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)}\frac{\vartheta\begin{bmatrix}\alpha+h\\ \beta+g\end{bmatrix}(0,\tau)\vartheta\begin{bmatrix}\alpha-h\\ \beta-g\end{bmatrix}(0,\tau)}{\vartheta\begin{bmatrix}1/2+h\\ 1/2+g\end{bmatrix}(0,\tau)\vartheta\begin{bmatrix}1/2-h\\ 1/2-g\end{bmatrix}(0,\tau)}\times\begin{cases}[-(2\sin(\pi g))^2] & \text{for } h=0\\ 1 & \text{for } h=\frac{1}{2}\end{cases}$$

where (g,h) take values $h=0$ for Möbius strip and the annulus diagrams with pure NN (N for Neumann) or DD (D for Dirichlet) boundary conditions whereas $h=\frac{1}{2}$ for ND, DN for annulus diagrams with boundary conditions. Further, $g=\frac{1}{2}$ whenever there is a reflection acting on the world-sheet oscillators in the trace, $g=0$ otherwise. Note that in the Möbius strip Ω acts on D directions with an additional reflection compared to N directions. To make sense of this formula also for $g=h=0$ one has to use

$$\lim_{\epsilon\to 0}\left[\frac{2\sin(\pi\epsilon)}{\vartheta\begin{bmatrix}1/2\\ 1/2+\epsilon\end{bmatrix}(0,\tau)}\right] = -\frac{1}{\eta^3(\tau)}\,. \tag{2.50}$$

The internal partition function of the Klein bottle is given by [28, 35, 37]

$$\mathcal{Z}^{\rm int}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \eta_{\alpha\beta}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)}\frac{\vartheta\begin{bmatrix}\alpha+h\\ \beta+2g\end{bmatrix}(0,\tau)\vartheta\begin{bmatrix}\alpha-h\\ \beta-2g\end{bmatrix}(0,\tau)}{\vartheta\begin{bmatrix}1/2+h\\ 1/2+2g\end{bmatrix}(0,\tau)\vartheta\begin{bmatrix}1/2-h\\ 1/2-2g\end{bmatrix}(0,\tau)}\times\begin{cases}[-(2\sin(2\pi g))^2] & \text{for } h=0\\ 16 & \text{for } h=\frac{1}{2}\end{cases}$$

where $h=0$ or $1/2$ for untwisted and twisted sectors, the factor of 16 coming from the 16 fixed points of the orbifold action. In the presence of a reflection in the trace $g=1/2$, otherwise $g=0$, but this does not have any effect, since only $2g$ appears.

[14] To be more precise, we should say that $\mathcal{Z}^{\rm int}_{\sigma,k}[{\alpha \atop \beta}]$ gives the internal partition function without the contribution from the zero modes that we split off.

JHEP11(2005)030

	Sector	Insertion	$\mathcal{Q}_{\sigma,k}$	$\gamma_{\sigma,k}$	$\vec{\mathbf{A}}_\sigma$
$\mathcal{K}$	$\sigma=(1)$	$k=0,1$	0		$\vec{0}$
	$\sigma=(\Theta)$	$k=0,1$	16		$\vec{0}$
$\mathcal{A}_{99}$	$\sigma=(ij)$	$k=0$	0	$\gamma_i\otimes\gamma_j^{-1}$	$(\vec{a}_iW_i\otimes\mathbf{1}_{32})\oplus(\mathbf{1}_{32}\otimes(-\vec{a}_jW_j))$
	$\sigma=(ij)$	$k=1$	-4	$\gamma_{\Theta i}\otimes\gamma_{\Theta j}^{-1}$	$(\vec{a}_iW_i\otimes\mathbf{1}_{32})\oplus(\mathbf{1}_{32}\otimes(-\vec{a}_jW_j))$
$\mathcal{A}_{55}$	$\sigma=(ab)$	$k=0$	0	$\gamma_a\otimes\gamma_b^{-1}$	$\vec{0}$
	$\sigma=(ab)$	$k=1$	-4	$\gamma_{\Theta a}\otimes\gamma_{\Theta b}^{-1}$	$\vec{0}$
$\mathcal{A}_{95}$	$\sigma=(ia)$	$k=0$	1	$\gamma_i\otimes\gamma_a^{-1}$	$\vec{a}_iW_i\otimes\mathbf{1}_{32}$
	$\sigma=(ia)$	$k=1$	1	$\gamma_{\Theta i}\otimes\gamma_{\Theta a}^{-1}$	$\vec{a}_iW_i\otimes\mathbf{1}_{32}$
$\mathcal{M}_9$	$\sigma=(i)$	$k=0$	0	$-\gamma_{\Omega i}^T\gamma_{\Omega i}^{-1}$	$2\vec{a}_iW_i$
	$\sigma=(i)$	$k=1$	-4	$-\gamma_{\Omega\Theta i}^T\gamma_{\Omega\Theta i}^{-1}$	$2\vec{a}_iW_i$
$\mathcal{M}_5$	$\sigma=(a)$	$k=0$	-4	$-\gamma_{\Omega a}^T\gamma_{\Omega a}^{-1}$	$\vec{0}$
	$\sigma=(a)$	$k=1$	0	$-\gamma_{\Omega\Theta a}^T\gamma_{\Omega\Theta a}^{-1}$	$\vec{0}$

Table 1: Information needed to compute the amplitudes.

The matrices $\gamma_{\sigma,k}$ stand for the operation on CP labels of the operators that appear in the trace and are given in table 1 for the different sectors. They are given by [37, 36]

$$\begin{aligned}
\gamma_i &= \mathbf{1}_{2N_i}\oplus\mathbf{0}_{32-2N_i}\,, \qquad \gamma_a = \mathbf{1}_{2N_a}\oplus\mathbf{0}_{32-2N_a}\,, \\
\gamma_{\Theta i} &= \mathrm{diag}(i\mathbf{1}_{N_i},-i\mathbf{1}_{N_i})\oplus\mathbf{0}_{32-2N_i}\,, \qquad \gamma_{\Theta a} = \mathrm{diag}(-i\mathbf{1}_{N_a},i\mathbf{1}_{N_a})\oplus\mathbf{0}_{32-2N_a}\,, \\
\gamma_{\Omega i} &= \sigma_{1N_i}\oplus\mathbf{0}_{32-2N_i}\,, \qquad \gamma_{\Omega a} = \sigma_{2N_a}\oplus\mathbf{0}_{32-2N_a}\,, \\
\gamma_{\Omega\Theta i} &= \sigma_{2N_i}\oplus\mathbf{0}_{32-2N_i}\,, \qquad \gamma_{\Omega\Theta a} = \sigma_{1N_a}\oplus\mathbf{0}_{32-2N_a}
\end{aligned} \tag{2.51}$$

with

$$\sigma_{1N_i} = \begin{pmatrix}\mathbf{0}_{N_i} & \mathbf{1}_{N_i}\\ \mathbf{1}_{N_i} & \mathbf{0}_{N_i}\end{pmatrix}, \qquad \sigma_{2N_i} = \begin{pmatrix}\mathbf{0}_{N_i} & i\mathbf{1}_{N_i}\\ -i\mathbf{1}_{N_i} & \mathbf{0}_{N_i}\end{pmatrix}. \tag{2.52}$$

In order to calculate the amplitude (2.48), we need the correlators (see also appendix A for a discussion)[15] for bosons on the torus

$$\langle Z_{\mathrm{qu}}(\nu_1)\bar{Z}_{\mathrm{qu}}(\nu_2)\rangle_{\mathcal{T}} = -\frac{\alpha'}{2}\ln\left|\frac{2\pi}{\vartheta_1'(0,\tau)}\vartheta_1\Big(\frac{\nu_1-\nu_2}{2\pi},\tau\Big)\right|^2 + \alpha'\frac{(\mathrm{Im}(\nu_1-\nu_2))^2}{4\pi\,\mathrm{Im}(\tau)}\,, \tag{2.53}$$

and for fermions on any world-sheet

$$\langle\Psi(\nu_1)\bar{\Psi}(\nu_2)\rangle_\sigma^{\alpha,\beta} = \langle\psi(\nu_1)\psi(\nu_2)\rangle_\sigma^{\alpha,\beta} = \frac{1}{2\pi}\frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\Big(\frac{\nu_1-\nu_2}{2\pi},\tau\Big)\vartheta_1'(0,\tau)}{\vartheta_1\Big(\frac{\nu_1-\nu_2}{2\pi},\tau\Big)\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}\,. \tag{2.54}$$

All correlators are obtained from correlators on the torus via the method of images [10], and the remaining ones are

$$\langle Z(\nu_1)\bar{Z}(\nu_2)\rangle_\sigma = \langle Z(\nu_1)\bar{Z}(\nu_2)\rangle_{\mathcal{T}} + \langle Z(\nu_1)\bar{Z}(I_\sigma(\nu_2))\rangle_{\mathcal{T}}\,,$$

[15] The derivative in $\vartheta_1'(0,\tau)$ is with respect to the first argument and not with respect to ν.

JHEP11(2005)030

$$
\begin{aligned}
\langle\tilde{\Psi}(\bar{\nu}_1)\bar{\Psi}(\nu_2)\rangle_\sigma^{\alpha,\beta} &= \langle\tilde{\psi}(\bar{\nu}_1)\psi(\nu_2)\rangle_\sigma^{\alpha,\beta} = i\langle\Psi(I_\sigma(\nu_1))\bar{\Psi}(\nu_2)\rangle_\sigma^{\alpha,\beta}\,, \\
\langle\tilde{\Psi}(\bar{\nu}_1)\bar{\tilde{\Psi}}(\bar{\nu}_2)\rangle_\sigma^{\alpha,\beta} &= \langle\tilde{\psi}(\bar{\nu}_1)\tilde{\psi}(\bar{\nu}_2)\rangle_\sigma^{\alpha,\beta} = \overline{\langle\Psi(\nu_1)\bar{\Psi}(\nu_2)\rangle_\sigma^{\alpha,\beta}}\,.
\end{aligned}
\tag{2.55}
$$

where the involution $I_\sigma(\nu)$ defines the type of world-sheet by identifying points on the torus. The involutions for the three types of one-loop diagrams are[16]

$$
I_{\mathcal{A}}(\nu) = I_{\mathcal{M}}(\nu) = 2\pi - \bar{\nu}\,, \qquad I_{\mathcal{K}}(\nu) = 2\pi - \bar{\nu} + \pi\tau\,. \tag{2.56}
$$

Again the purely holomorphic or anti-holomorphic correlators vanish. One can then use

$$
\sum_{\substack{\alpha,\beta\\ \text{even}}} \eta_{\alpha\beta}\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(\nu,\tau)^2\vartheta\begin{bmatrix}\alpha+h\\ \beta+g\end{bmatrix}(0,\tau)\vartheta\begin{bmatrix}\alpha-h\\ \beta-g\end{bmatrix}(0,\tau) = \vartheta_1(\nu,\tau)^2\vartheta\begin{bmatrix}1/2+h\\ 1/2+g\end{bmatrix}(0,\tau)\vartheta\begin{bmatrix}1/2-h\\ 1/2-g\end{bmatrix}(0,\tau) \tag{2.57}
$$

to evaluate the sum over spin structures. In doing so, we define

$$
\begin{aligned}
\mathcal{Q}_{\sigma,k} &= \sum_{\substack{\alpha\beta\\ \text{even}}} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)}\mathcal{Z}^{\text{int}}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\langle\Psi(\nu_1)\bar{\Psi}(\nu_2)\rangle_\sigma^{\alpha,\beta}\langle\psi(\nu_1)\psi(\nu_2)\rangle_\sigma^{\alpha,\beta} \\
&= \begin{cases} \left.\begin{matrix} -(2\sin(\pi g))^2 & \text{for } h=0 \\ 1 & \text{for } h=\frac{1}{2}\end{matrix}\right\} & \text{for } \mathcal{A},\mathcal{M} \\ \quad 0 & \text{for } \mathcal{K}_1 \\ \quad 16 & \text{for } \mathcal{K}_\Theta \end{cases}\,.
\end{aligned}
\tag{2.58}
$$

Note that the dependence on ν_i and τ drops out. Furthermore, $\mathcal{Q}_{\sigma,k}$ is zero for the $\mathcal{N}=4$ subsector of the theory, which means that

$$
\mathcal{K}_1^{(0)} = \mathcal{A}_{99}^{(0)} = \mathcal{M}_9^{(0)} = 0\,, \qquad \mathcal{K}_1^{(1)} = \mathcal{A}_{55}^{(0)} = \mathcal{M}_5^{(1)} = 0\,. \tag{2.59}
$$

These are the contributions of the original type-I theory compactified on a torus, and its T-dual along the $\mathbb{T}^4$. Furthermore, since the matrix $\gamma_{\Theta a}$ is traceless, the annulus diagrams with $k=1$ and at least one boundary on a D5-brane, i.e. $\mathcal{A}_{55}^{(1)} + \mathcal{A}_{95}^{(1)} + \mathcal{A}_{59}^{(1)}$ also do not contribute. This leaves us with the non-vanishing contribution from diagrams

$$
\mathcal{K}_\Theta^{(0)} + \mathcal{K}_\Theta^{(1)} + \mathcal{A}_{99}^{(1)} + \mathcal{A}_{95}^{(0)} + \mathcal{A}_{59}^{(0)} + \mathcal{M}_9^{(1)} + \mathcal{M}_5^{(0)}\,. \tag{2.60}
$$

To be explicit, in table 1 we list all the quantities $\mathcal{Q}_{\sigma,k}$, $\gamma_{\sigma,k}$ and the Wilson lines $\vec{\mathbf{A}}_\sigma$ that are relevant to compute the contributions.

This leaves the correlators of the bosonic fields as the only piece that depends on the positions ν_i of the vertex operators. The integral is evaluated via

$$
\begin{aligned}
\int_{\mathcal{F}_\sigma} & d^2\nu_1 d^2\nu_2\Big[\langle\bar{\partial}Z(\bar{\nu}_1)\bar{\partial}\bar{Z}(\bar{\nu}_2)\rangle_\sigma - \langle\bar{\partial}Z(\bar{\nu}_1)\partial\bar{Z}(\nu_2)\rangle_\sigma + \text{c.c.}\Big] = \\
&= -2\pi^4 c_\sigma^2\frac{T_2}{U_2}|n+mU|^2\alpha' + \pi^3 c_\sigma^2 t\alpha' \\
&= \begin{cases} -2\pi^4\frac{T_2}{U_2}|n+mU|^2\alpha' + \pi^3\alpha' t & \text{for } \mathcal{A},\mathcal{M} \\ -8\pi^4\frac{T_2}{U_2}|n+mU|^2\alpha' + 4\pi^3\alpha' t & \text{for } \mathcal{K}\end{cases}\,,
\end{aligned}
\tag{2.61}
$$

[16]Compare also figure 3.

JHEP11(2005)030

where the first contribution comes from the zero modes given in formula (2.49), where also the constants c_σ are introduced.[17] In order to evaluate the quantum part of (2.61), i.e. the second contribution, we made use of the fact that a function $f(\nu)$ that is periodic on the covering torus satisfies [10]

$$\int_{\mathcal{F}_\sigma} d^2\nu \left[\partial_\nu f(\nu) - \partial_{\bar\nu} f(I_\sigma(\nu))\right] = \int_{\mathcal{F}_\mathcal{T}} d^2\nu\, \partial_\nu f(\nu) = 0\,. \tag{2.62}$$

To evaluate the trace and KK sum, it is useful to regularize the integral over the t with a UV cutoff Λ and introduce new variables

$$l = \frac{1}{e_\sigma t} = \begin{cases} 1/t & \text{for } \mathcal{A} \\ 1/(4t) & \text{for } \mathcal{M}, \mathcal{K} \end{cases}, \tag{2.63}$$

where we introduced yet another constant e_σ, also listed in (C.11). With this we get

$$\begin{aligned}
&\int_{1/(e_\sigma\Lambda^2)}^\infty \frac{dt}{t^4} \sum_{\vec n=(n,m)^T} \left[e^{-\pi\vec n^T G\vec n t^{-1}} e^{2\pi i\vec{\mathbf{A}}_\sigma\cdot\vec n}\left[-2\pi^4 c_\sigma^2\frac{T_2}{U_2}|n+mU|^2\alpha' + \pi^3 c_\sigma^2 t\alpha'\right]\right] = \\
&= \int_0^\infty dl \sum_{\vec n=(n,m)^T}{}' \left[e^{-\pi\vec n^T G\vec n e_\sigma l} e^{2\pi i\vec{\mathbf{A}}_\sigma\cdot\vec n}\left[-2\pi^4 c_\sigma^2 e_\sigma^3 l^2\frac{T_2}{U_2}|n+mU|^2\alpha' + \pi^3 c_\sigma^2 e_\sigma^2 l\alpha'\right]\right] + \\
&\quad +\pi^3\alpha' c_\sigma^2 e_\sigma^2 \int_0^{\Lambda^2} dl\, l + \cdots \\
&= \frac{1}{2}\pi^3\alpha' c_\sigma^2 e_\sigma^2\Lambda^4 - 3\pi\alpha' c_\sigma^2 T_2^{-2} E_2(\mathbf{A}_\sigma, U) + \cdots\,,
\end{aligned} \tag{2.64}$$

where the prime at the sum indicates that one has to leave out the term with $(n,m) = (0,0)$. Terms that go to zero in the limit $\Lambda \to \infty$ have been dropped (indicated by the ellipsis). Moreover, we abbreviated

$$\begin{aligned}
E_s(A,U) &= \sum_{\vec n=(n,m)^T}{}' \frac{U_2^s e^{2\pi i\vec n\cdot\vec a}}{|n+mU|^{2s}} \\
&= \sum_{\vec n=(n,m)^T}{}' \frac{U_2^s}{|n+mU|^{2s}} \exp\left[2\pi i\frac{A(n+m\bar U) - \bar A(n+mU)}{U-\bar U}\right].
\end{aligned} \tag{2.65}$$

This function reduces to the ordinary non-holomorphic Eisenstein series $E_s(U)$ when $A = 0$. In (2.64) we have used a boldface $\mathbf{A}_\sigma$ (without vector arrow) referring to matrix valued open string scalars, i.e. $\mathbf{A}_\sigma$ is defined by replacing $\vec a_i$ by A_i everywhere in table 1 (recall the relation between them, given in (2.2)). The terms involving the UV cutoff Λ drop out after summing over all diagrams due to tadpole cancellation. We are thus left with the expression

$$\langle V_{Z\bar Z}^{(0,0)} V_{Z\bar Z}^{(0,0)}\rangle_\sigma = (p_1\cdot p_2)\alpha'\frac{V_4}{(4\pi^2\alpha')^2}\frac{3c_\sigma^2\pi}{8T_2}\sum_k \mathrm{tr}\left[E_2(\mathbf{A}_\sigma, U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right] + \mathcal{O}(\delta^2)\,. \tag{2.66}$$

[17]For reference, we also collect these in (C.11), together with other constants that will be introduced shortly.

We can then evaluate the relevant traces for all the diagrams that appear in (2.60) and find

$$\begin{aligned}
&\sum_\sigma c_\sigma^2 \sum_{k=0,1} \mathrm{tr}\left[E_2(\mathbf{A}_\sigma, U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right] = \\
&\quad = 4E_2(0,U)\left[\mathcal{Q}_{(\Theta),0} + \mathcal{Q}_{(\Theta),1}\right] + \sum_{i,j} \mathrm{tr}\left[E_2(\mathbf{A}_{(ij)}, U)\gamma_{(ij),1}\mathcal{Q}_{(ij),1}\right] + \\
&\qquad +2\sum_{i,a} \mathrm{tr}\left[E_2(\mathbf{A}_{(i)}, U)\gamma_{(ia),0}\mathcal{Q}_{(ia),0}\right] + \sum_i \mathrm{tr}\left[E_2(\mathbf{A}_{(i)}, U)\gamma_{(i),1}\mathcal{Q}_{(i),1}\right] + \\
&\qquad + \sum_a \mathrm{tr}\left[E_2(0,U)\gamma_{(a),0}\mathcal{Q}_{(a),0}\right] \\
&\quad = -4\sum_{i,j} N_i N_j \Big[E_2(A_i - A_j, U) + E_2(-A_i + A_j, U) - \\
&\qquad\qquad\qquad - E_2(A_i + A_j, U) - E_2(-A_i - A_j, U)\Big] + \\
&\qquad +2\cdot 32 \sum_i N_i [E_2(A_i, U) + E_2(-A_i, U)] - \\
&\qquad -4\sum_i N_i [E_2(2A_i, U) + E_2(-2A_i, U)]\,,
\end{aligned} \tag{2.67}$$

where the contributions from the Klein bottle and the 5-brane Möbius strip diagrams, which involve $E_2(0,U)$, just cancel out. We denote this quantity as

$$\mathcal{E}_2(A_i, U) = \sum_\sigma c_\sigma^2 \sum_{k=0,1} \mathrm{tr}\left[E_2(\mathbf{A}_\sigma, U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right]. \tag{2.68}$$

Putting everything together we end up with

$$\langle V_{S_2'} V_{S_2'}\rangle = -i(p_1\cdot p_2)\alpha' \frac{V_4}{(4\pi^2\alpha')^2}\frac{3g_c^2\alpha'^{-4}e^{-\Phi}}{8\sqrt{2}}\frac{1}{(S_0' - \bar{S}_0')^3}\mathcal{E}_2(A_i, U)\,. \tag{2.69}$$

From this we can read off the one-loop correction to the kinetic term of S' after performing a Weyl rescaling to the Einstein frame. In the one-loop term (2.69) this just leads to an additional factor of

$$\frac{e^{2\Phi}}{\mathcal{V}_{\mathrm{K3}}\sqrt{G}} = \frac{ie^{\Phi}}{\sqrt{2}\pi(S - \bar{S})}\,. \tag{2.70}$$

There is one further complication, the one-loop correction to the Einstein-Hilbert term calculated in [10]. In the case at hand, i.e. in the presence of Wilson line moduli, the corrected Einstein-Hilbert term is

$$\frac{1}{2}\left(e^{-2\Phi}\mathcal{V}_{\mathrm{K3}}\sqrt{G} + \frac{\tilde{c}}{\sqrt{G}}\mathcal{E}_2(A_i, U)\right)R\,, \tag{2.71}$$

with $\tilde{c}$ a constant, whose value is not important for us at the moment. Due to the presence of a kinetic term for S' already at sphere level this produces extra corrections to the one-loop corrected kinetic term in Einstein frame. Performing a Weyl rescaling to the four-dimensional Einstein frame, i.e.

$$g_{\mu\nu} \xrightarrow{\text{Weyl}} \left(e^{-2\Phi}\mathcal{V}_{\mathrm{K3}}\sqrt{G} + \frac{\tilde{c}}{\sqrt{G}}\mathcal{E}_2(A_i, U)\right)^{-1} g_{\mu\nu}\,, \tag{2.72}$$

and expanding in the dilaton, the term proportional to $\mathcal{E}_2(A_i,U)$ contributes to the sphere level kinetic term, cf. the first line of (2.16), and thus changes the prefactor of the term arising from (2.69).[18] Another possible source of modification of the overall factor is if the variable S' is corrected again at one-loop, i.e. in addition to the disk level correction present in (2.3). A similar effect for S was noted in [10]. In analogy to that case one might expect a correction

$$\mathrm{Im}(S') \;\rightarrow\; \mathrm{Im}(S') + \hat{c}\,\mathcal{E}_2(A_i,U)\,\mathrm{Im}(S)^{-1} \tag{2.73}$$

that would also modify the coefficient of the term proportional to $\mathcal{E}_2(A_i,U)$ in the kinetic term for S' in the Einstein frame. Given that we do not know the constant $\hat{c}$ exactly, we leave the overall factor in the one-loop correction to the kinetic term of S' open for the moment and come back to it in the next section. Making the replacements[19]

$$V_4 \;\rightarrow\; d^4x\sqrt{-g}\,, \qquad (p_1\cdot p_2)g_c^2\alpha'^{-4} \;\rightarrow\; \partial_\mu S_2'\partial^\mu S_2'\,, \tag{2.74}$$

where we skipped some numerical factors, and using the fact that

$$K_{S'\bar{S}'}\partial_\mu S'\partial^\mu \bar{S}' = K_{S'\bar{S}'}(\partial_\mu S_1'\partial^\mu S_1' + \partial_\mu S_2'\partial^\mu S_2')\,, \tag{2.75}$$

we finally read off

$$K_{S'\bar{S}'} \sim \frac{\mathcal{E}_2(A_i,U)}{(S_0'-\bar{S}_0')^3(S-\bar{S})}\,. \tag{2.76}$$

2.6 One-loop Kähler potential and prepotential

To find the Kähler potential that reproduces our one-loop correction to the Kähler metric (2.76), we can make use of the fact that for $\mathcal{N}=2$ supersymmetry the Kähler potential is given by a prepotential according to the relation (2.14). In particular, it must be possible to express the correction to the Kähler potential as a correction to the argument of the logarithm of the "classical" Kähler potential (2.12). An obvious candidate that reproduces (2.76) up to higher order terms is

$$\begin{aligned} K = &-\ln\Big[(S-\bar{S})(S'-\bar{S}')(U-\bar{U})\Big] - \\ &-\ln\left[1 - \frac{1}{8\pi}\sum_i \frac{N_i(A_i-\bar{A}_i)^2}{(S'-\bar{S}')(U-\bar{U})} - \sum_i \frac{c\,\mathcal{E}_2(A_i,U)}{(S-\bar{S})(S'-\bar{S}')}\right], \end{aligned} \tag{2.77}$$

where $\mathcal{E}_2(A_i,U)$ is given in (2.68) and (2.65). The expression (2.77) contains terms of all orders in the string coupling, when the logarithm is expanded. In appendix C we verify that this Kähler potential also correctly reproduces all the other one-loop corrections to the metrics of $\{U,S',A_i\}$ at the relevant order in the string coupling. From the point of view of finding the effective lagrangian that reproduces our string amplitudes, this justifies the use of (2.77) as the one-loop corrected Kähler potential of the $\mathbb{Z}_2$ orientifold. Thus, we consider (2.77) one of the main results of this paper.

[18] More concretely, it would cancel the contribution to the correlator (2.61) coming from the fluctuations and only leave those from the zero modes. We will make this more precise for the case of the modulus U in appendix C, cf. equation (C.33).

[19] The power of α' in the second replacement is chosen to give the right dimension, given that S' is defined to be dimensionless and $g_c \sim \alpha'^2$.

JHEP11(2005)030

The constant c is not easily determined directly from the present calculation of the one-loop correction to the Kähler metric. In addition to the issues we explained in the previous section, one would have to know the relative normalization between the tree level and one loop amplitudes. Instead, we will use the relation (2.15) to fix it to the value $c = 1/(128\pi^6)$. This is summarized in appendix D.

To make sure that (2.77) is consistent with $\mathcal{N} = 2$ supersymmetry, we now express the argument of the logarithm in terms of a prepotential as in (2.14). Expanding the prepotential perturbatively into

$$\mathcal{F}(S, S', U, A_i) = \mathcal{F}^{(0)}(S, S', U, A_i) + \mathcal{F}^{(1)}(U, A_i)\,, \tag{2.78}$$

the classical term is given in (2.13). To find $\mathcal{F}^{(1)}(U, A_i)$ we have to convert the correction of the argument of the logarithm in (2.77) into a prepotential. This means we must recast $U_2\mathcal{E}_2(A_i, U)$ in the form of the argument of equation (2.14). To do so, note that[20]

$$\begin{aligned}(U-\bar{U})E_2(A,U) \;=\; & 4i\pi^4\left(\frac{1}{90}U_2^3 - \frac{1}{3}U_2A_2^2 + \frac{2}{3}A_2^3 - \frac{1}{3}\frac{A_2^4}{U_2}\right) + \\ & + i\pi\Big[Li_3(e^{2\pi iA}) + 2\pi A_2 Li_2(e^{2\pi iA}) + \text{c.c.}\Big] + \\ & + 2i\pi^2\sum_{m>0}\Big[(mU_2 - A_2)\,Li_2(e^{2\pi i(mU-A)}) + \\ & \qquad + (mU_2 + A_2)\,Li_2(e^{2\pi i(mU+A)}) + \text{c.c.}\Big] + \\ & + i\pi\sum_{m>0}\Big[Li_3(e^{2\pi i(mU-A)}) + Li_3(e^{2\pi i(mU+A)}) + \text{c.c.}\Big]\,,\end{aligned} \tag{2.79}$$

where the polylogarithms are defined in (B.5). With the help of

$$\frac{d}{dx}Li_n(x) = \frac{1}{x}Li_{n-1}(x)\,, \tag{2.80}$$

it follows that

$$(U-\bar{U})E_2(A,U) = -\frac{4i\pi^4}{3}\frac{A_2^4}{U_2} + 2h - 2\bar{h} - (U-\bar{U})(\partial_U h + \partial_{\bar{U}}\bar{h}) - (A-\bar{A})(\partial_A h + \partial_{\bar{A}}\bar{h}) \tag{2.81}$$

with

$$\begin{aligned}h(A,U) \;=\; & \frac{\pi^4}{2}\left[\frac{1}{90}U^3 - \frac{1}{3}UA^2 + \frac{2}{3}A^3\right] + \frac{i\pi}{2}Li_3(e^{2\pi iA}) + \\ & + \frac{i\pi}{2}\sum_{m>0}\Big[Li_3(e^{2\pi i(mU-A)}) + Li_3(e^{2\pi i(mU+A)})\Big]\,.\end{aligned} \tag{2.82}$$

Note that the extra term of the form A_2^4/U_2 in (2.81) drops out when summing over diagrams in (2.67) and using the anomaly constraint (C.24). We can write

$$(U-\bar{U})\mathcal{E}_2(A_i,U) = 2f - 2\bar{f} - (U-\bar{U})(\partial_U f + \partial_{\bar{U}}\bar{f}) - \sum_i (A_i - \bar{A}_i)(\partial_{A_i} f + \partial_{\bar{A}_i}\bar{f}) \tag{2.83}$$

[20]See appendix B for the derivation of this formula and more details on the function $E_2(A, U)$. Also, in order to avoid cluttering the formulas with too many indices we give them only for a single $A = A_1 + iA_2$. The generalization to several A_i is straightforward.

with

$$
\begin{aligned}
f(A_i,U) = -4\sum_{i,j} N_iN_j\Big[h(A_i-A_j,U)+h(-A_i+A_j,U)-h(A_i+A_j,U)-\\
-h(-A_i-A_j,U)\Big]+64\sum_i N_i[h(A_i,U)+h(-A_i,U)]-\\
-4\sum_i N_i[h(2A_i,U)+h(-2A_i,U)]\,. \qquad (2.84)
\end{aligned}
$$

Thus, the one-loop correction to the prepotential is given by

$$
\mathcal{F}^{(1)}(A_i,U) = c\, f(A_i,U)\,, \qquad (2.85)
$$

where the constant c is given in (D.7). This form is consistent with the prepotentials calculated in the case of the heterotic string. The duality to type-I was discussed in [19]. In the case of the heterotic string, the prepotential generally has the form of a sum of cubic monomials in the fields with polylogarithms, plus a universal term involving the coefficient $\zeta(3)$, which is recovered here at $A=0$ via $Li_k(1)=\zeta(k)$.[21] We view the existence of the prepotential (2.85) as strong support for the validity of (2.77). Moreover, as the prepotential (2.78) does not receive any further perturbative corrections, the result (2.77) holds to all orders of perturbation theory.

3. The $\mathcal{N}=1$ orientifold $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$

We now extend the previous analysis to orientifold compactifications with $\mathcal{N}=1$ supersymmetry, treating first the case $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ [39] (see also [40]). Its orbifold group is generated by two reflection operators, each non-trivial element leaving a 2-torus in $\mathbb{T}^6=\mathbb{T}^2_1\times\mathbb{T}^2_2\times\mathbb{T}^2_3$ invariant. We write the orbifold group $\mathbb{Z}_2\times\mathbb{Z}_2=\{1,\Theta_1,\Theta_2,\Theta_3=\Theta_1\Theta_2\}$, the eigenvalues $\exp(2\pi i\vec{v}_I)$ of the elements being characterized via the three vectors $\vec{v}_I$

$$
\vec{v}_1=\Big(0,\frac{1}{2},-\frac{1}{2}\Big)\,,\qquad \vec{v}_2=\Big(-\frac{1}{2},0,\frac{1}{2}\Big)\,,\qquad \vec{v}_3=\vec{v}_1+\vec{v}_2=\Big(-\frac{1}{2},\frac{1}{2},0\Big)\,. \qquad (3.1)
$$

We label the two-torus $\mathbb{T}^2_I$ by the same index as the element Θ_I that leaves it invariant. Similarly $\mathbb{T}^4_I$ is the four-torus reflected by Θ_I. The model includes three sets of O5-planes and D5-branes, each wrapped along one torus and labelled $\mathrm{O5}_I$, $\mathrm{D5}_I$, and O9-planes and D9-branes wrapping the entire internal space.

3.1 The classical lagrangian

The closed string moduli space of the $\mathcal{N}=1$ orientifold on $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ consists of three copies of the moduli space of a two-torus, plus the universal axio-dilaton multiplet and the blow-up modes from the 48 twisted sectors. In all, the Hodge numbers of the space are $(h^{(1,1)},h^{(2,1)})=(3,51)$. We are only interested in untwisted moduli and ignore the 48 twisted scalars in the following. Besides the neutral closed string fields, the moduli space

[21] Compare e.g. equation (4.25) of [38].

also includes the D9-brane Wilson lines along the three tori $\mathbb{T}^2_I$ and the position scalars and Wilson lines of the D5-branes. The latter we also set to zero. We again allow for a number of D9-brane stacks with three complex Wilson lines A^I_i each, and define the scalars[22]

$$\begin{aligned} T^I &= \frac{1}{\sqrt{8\pi^2}}(C^I_{45} + ie^{-\Phi}\sqrt{G^I}) + \frac{1}{8\pi}\sum_i N_i(U^I(a^I_i)^2_4 - (a^I_i)_4(a^I_i)_5) \\ &= T^I_0 + \frac{1}{8\pi}\sum_i N_i A^I_i \frac{A^I_i - \bar{A}^I_i}{U^I - \bar{U}^I}\,, \\ U^I &= \frac{1}{G^I_{44}}(G^I_{45} + i\sqrt{G^I})\,, \qquad A^I_i = U^I(a^I_i)_4 - (a^I_i)_5\,, \end{aligned} \tag{3.2}$$

with G^I_{mn} being the metric on each $\mathbb{T}^2_I$, C^I_{45} the RR 2-form, and the $(a^I_i)_m$ the components of the Wilson lines of the D9-brane stack labelled by i, $I \in \{1,2,3\}$, see also [41]. The dilaton S is defined as in the $\mathcal{N}=2$ case above. The classical Kähler potential is given by

$$\begin{aligned} K^{(0)}_{\mathbb{Z}^2_2} &= -\ln(S-\bar{S}) - \sum_{I=1}^{3}\ln\left[(T^I_0 - \bar{T}^I_0)(U^I - \bar{U}^I)\right] \\ &= -\ln(S-\bar{S}) - \sum_{I=1}^{3}\ln\left[(T^I - \bar{T}^I)(U^I - \bar{U}^I) - \frac{1}{8\pi}\sum_i N_i(A^I_i - \bar{A}^I_i)^2\right]. \end{aligned} \tag{3.3}$$

The classical gauge kinetic functions are

$$f^{(0)}_{\mathrm{D9}} = -iS\,, \qquad f^{(0)}_{\mathrm{D5}_I} = -iT^I_0\,. \tag{3.4}$$

All these expressions can be derived completely analogously to section 2.1, via dimensional reduction of the type-I plus Born-Infeld supergravity lagrangian from ten dimensions on a product of three tori. For the most part, this model can be thought of as the direct sum of three copies of the $\mathcal{N}=2$ compactification on K3$\times\mathbb{T}^2$: three non-trivial elements of the orbifold group act on three $\mathbb{T}^2_I$ with moduli $\{T^I, U^I, A^I_i\}$, and there are three sets of D5_I-branes instead of a single one. Only the multiplet S and the D9-branes are universal. The effective lagrangian is then very similar to (2.11) where one only needs to sum over the three tori in the form

$$\begin{aligned} \kappa_4^2\mathcal{L}_{4d} = &\frac{1}{2}R + \frac{\partial_\mu S\partial^\mu\bar{S}}{(S-\bar{S})^2} - \frac{1}{4}\kappa_4^2\,\mathrm{Im}(S)\,\mathrm{tr}\,\mathcal{F}^2_{\mathrm{D9}} + \\ &+\sum_{I=1}^{3}\left[\frac{\partial_\mu U^I\partial^\mu\bar{U}^I}{(U^I-\bar{U}^I)^2} + \frac{|\partial_\mu T^I_0 + \frac{1}{8\pi}\sum_i N_i((a^I_i)_4\partial_\mu(a^I_i)_5 - (a^I_i)_5\partial_\mu(a^I_i)_4)|^2}{(T^I_0-\bar{T}^I_0)^2} + \right. \\ &\left. + \frac{\sum_i N_i|U^I\partial_\mu(a^I_i)_4 - \partial_\mu(a^I_i)_5|^2}{4\pi(U^I-\bar{U}^I)(T^I_0-\bar{T}^I_0)} - \frac{1}{4}\kappa_4^2\,\mathrm{Im}(T^I_0)\,\mathrm{tr}\,\mathcal{F}^2_{\mathrm{D5}_I}\right]. \end{aligned}$$

The classical Kähler metric can also be read off from (2.16) since it factorizes into the three tori.

[22] Note that the normalization used here for the tree level part of T^I differs from [20]. Here, we choose the normalization of $\sqrt{G^I}$ such that the factors are identical to the $\mathcal{N}=2$ case. In addition, so as not to overload the notation, we always use "4" and "5" for the internal directions, even though for $I=2$ ($I=3$), they are of course "6" and "7" ("8" and "9").

JHEP11(2005)030

However, there are important differences compared to the $\mathcal{N}=2$ case. In particular, one cannot deduce the exact form of the Kähler potential as a logarithm of a corrected argument such as in (2.14), but only its perturbative expansion

$$K = K^{(0)} + \sum_{n=1}^{\infty} K^{(n)} \tag{3.5}$$

where for $n \geq 1$, $n+1$ denotes the power of e^{Φ} in $K^{(n)}$, i.e. $K^{(n)} \propto e^{(n+1)\Phi}$. The sum starts at $n+1=2$ because the classical piece already includes the disk diagrams. Explicitly, we will find three terms that behave like $(S-\bar{S})^{-1}(T^I-\bar{T}^I)^{-1}$ at the level $n=2$, just as for the $\mathcal{N}=2$ model, plus three terms of the form $(T^I-\bar{T}^I)^{-1}(T^J-\bar{T}^J)^{-1}$.

3.2 One-loop amplitudes

The vertex operators for the untwisted moduli of the three $\mathbb{T}^2_I$ are identical to those derived for the torus in the $\mathcal{N}=2$ model in section 2.3, and can be read off from (2.34) by simply adding the label I for the three tori. This follows immediately from the fact that the orbifold projection does not constrain the metric or the Wilson lines, except for imposing conditions on the CP matrices λ. Therefore, the form (2.17) of the world-sheet lagrangian is formally unmodified here, only using a different λ.

We can now follow the steps of section 2.5 to compute the correlators $\langle V_{T_2^I} V_{T_2^I}\rangle$, i.e. the equivalent of (2.45). It will give us the Kähler potential upon integration just as in the case of the $\mathcal{N}=2$ model.

To evaluate the one-loop amplitudes, it is most useful to split all contributions into those which are repeated copies of the ones that already appeared in the computation of section 2.5, and extra pieces. The Klein bottle, as all other diagrams, now includes three non-trivial insertions Θ_I in the trace from expanding the orbifold projector

$$\mathcal{P} = \frac{1}{2}(1+\Theta_1)\frac{1}{2}(1+\Theta_2) = \frac{1}{4}(1+\Theta_1+\Theta_2+\Theta_3)\,, \tag{3.6}$$

and a sum over $3\times 16 = 48$ twisted sectors at the fixed points of the three Θ_I. The Möbius strip diagrams now have boundaries on the D9- and the D5$_I$-branes and (3.6) inserted. For the annulus diagrams, there are 99, diagrams, three sets of 5_I5_I diagrams, and three sets of 95_I diagrams. In addition there are three 5_I5_J diagrams, $I \neq J$, that do not have any analog in the $\mathcal{N}=2$ case. In all, we have the one-loop diagrams

$$\sum_{\hat{I}=0}^{3}\left[\mathcal{K}_1^{(\hat{I})} + \sum_{J=1}^{3}\mathcal{K}_{\Theta_J}^{(\hat{I})} + \mathcal{M}_9^{(\hat{I})} + \sum_{J=1}^{3}\mathcal{M}_{5_J}^{(\hat{I})} + \mathcal{A}_{99}^{(\hat{I})} + \sum_{J=1}^{3}\left[\mathcal{A}_{5_J5_J}^{(\hat{I})} + \mathcal{A}_{95_J}^{(\hat{I})} + \mathcal{A}_{5_J9}^{(\hat{I})} + \sum_{K\neq J}\mathcal{A}_{5_J5_K}^{(\hat{I})}\right]\right], \tag{3.7}$$

where we have defined a label $\hat{I} \in \{0,1,2,3\}$, which refers to the insertion of either Θ_I or the identity into the trace.[23] The computation of these amplitudes is now in principle

[23] Once we introduce Wilson lines, we would actually need to use a notation as in (2.47). To keep the notation reasonably compact, we refrain from making that explicit.

identical to that of the $\mathcal{N}=2$ case in section 2.5. The internal partition functions of the annulus and Möbius diagrams read

$$\mathcal{Z}^{\rm int}_{\sigma,\hat{I}}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \eta_{\alpha\beta}\prod_{I=1}^{3}\frac{\vartheta\begin{bmatrix}\alpha+h_I\\ \beta+g_I\end{bmatrix}(0,\tau)}{\vartheta\begin{bmatrix}1/2+h_I\\ 1/2+g_I\end{bmatrix}(0,\tau)}\times\begin{cases}(2\sin(\pi g_I)) & \text{for } h_I=0\\ 1 & \text{for } h_I=\frac{1}{2}\end{cases}. \tag{3.8}$$

The values of g_I are 0 or $\pm\frac{1}{2}$ for a trace with either the identity or a reflection of $\mathbb{T}^2_I$ inserted, the sign determined by using (3.1). The assignment is partly reversed for the 5-brane Möbius strip, where world-sheet parity Ω acts on a field with Dirichlet (D) boundary conditions with an extra reflection compared to its operation with Neumann (N) boundary conditions. The h_I are all 0 in the Möbius diagrams, and in the annulus they are 0 for an open string sector with NN or DD boundary conditions, or $\pm\frac{1}{2}$ for ND and DN boundary conditions. The partition function of the Klein bottle is

$$\mathcal{Z}^{\rm int}_{\sigma,\hat{I}}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \eta_{\alpha\beta}\prod_{I=1}^{3}\frac{\vartheta\begin{bmatrix}\alpha+h_I\\ \beta+2g_I\end{bmatrix}(0,\tau)}{\vartheta\begin{bmatrix}1/2+h_I\\ 1/2+2g_I\end{bmatrix}(0,\tau)}\times\begin{cases}(2\sin(2\pi g_I)) & \text{for } h_I=0\\ 16 & \text{for } h_I=\frac{1}{2}\end{cases}. \tag{3.9}$$

Again, h_I and g_I are 0 or $\pm\frac{1}{2}$ for untwisted and twisted sectors and insertions of reflections on the three $\mathbb{T}^2_I$, respectively. For $g=h=0$, one again has to interpret these formulas in the sense of the limit (2.50). Note that for the Klein bottle, the presence of a reflection does not affect the partition function of the internal string oscillators, since only $2g_I$ appears. The reflection does, however, determine whether the spectrum of KK states invariant under the insertion involves winding or momentum modes (in the open channel).

Using the identity (2.57) one can evaluate the sum over spin structures, and obtains the numerical coefficients $\mathcal{Q}_{\sigma,\hat{I}}$ as defined in (2.58). The supersymmetric solution for the orientifold action on the CP matrices was found in [39]. It gives a maximal gauge group $\mathrm{Sp}(8)^4$ of rank 32, each factor referring to one of the four types of D9- or D5$_I$-branes. For the moment we just need the properties[24]

$$\gamma_{\Omega\Theta_I 9} = -\gamma^T_{\Omega\Theta_I 9}\,,\qquad \gamma_{\Omega 5_I} = -\gamma^T_{\Omega 5_I}\,,\qquad \gamma_{\Omega\Theta_I 5_J} = -\gamma^T_{\Omega\Theta_I 5_J}\quad\text{for}\quad I\neq J\,, \tag{3.10}$$

and that $\gamma_{\Theta_I 5_J}$ is traceless. Thus, the traces in the Möbius strip diagrams behave like those for the matrices $\gamma_{\Omega\Theta i}$ or $\gamma_{\Omega a}$ in (2.51).

We now discuss the evaluation of all the diagrams of (3.7) piece by piece. First note that the $\mathcal{N}=4$ subsectors do not contribute because, just as in the $\mathcal{N}=2$ case, the relevant factors $\mathcal{Q}_{\sigma,\hat{I}}$ vanish as in table 1. In other words, there are four sets of vanishing amplitudes

$$\mathcal{K}^{(0)}_1 = \mathcal{M}^{(0)}_9 = \mathcal{A}^{(0)}_{99} = 0\,,\qquad \mathcal{K}^{(I)}_1 = \mathcal{M}^{(I)}_{5_I} = \mathcal{A}^{(0)}_{5_I 5_I} = 0\,. \tag{3.11}$$

These are just the diagrams of type-I string theory compactified on a torus, respectively its T-dual versions (with 4 T-dualities along $\mathbb{T}^4_I$). Next there are three copies of the $\mathcal{N}=2$ diagrams computed in section 2.5, one for each value of I,

$$\mathcal{K}^{(0)}_{\Theta_I} + \mathcal{K}^{(I)}_{\Theta_I} + \mathcal{M}^{(I)}_9 + \mathcal{M}^{(0)}_{5_I} + \mathcal{A}^{(I)}_{99} + \mathcal{A}^{(I)}_{5_I 5_I} + \mathcal{A}^{(0)}_{95_I} + \mathcal{A}^{(0)}_{5_I 9} + \mathcal{A}^{(I)}_{95_I} + \mathcal{A}^{(I)}_{5_I 9}\,. \tag{3.12}$$

[24]The matrices are called M_i or N_1 in [39] where they are defined in a table on page 16. See also equations (4.5) and (4.6).

The amplitudes $\mathcal{A}^{(I)}_{5_I5_I}$ and $\mathcal{A}^{(I)}_{95_I}$ actually do not contribute due to the matrices γ being traceless. Further, (3.10) implies the cancellation of the Klein bottle diagrams with the 5-brane Möbius strip diagrams as in (2.67) such that the final set of amplitudes, that are the analogs of the $\mathcal{N}=2$ model of the last section, is

$$\sum_{I=1}^{3}\left[\mathcal{M}^{(I)}_9+\mathcal{A}^{(I)}_{99}+\mathcal{A}^{(0)}_{95_I}+\mathcal{A}^{(0)}_{5_I9}\right]. \tag{3.13}$$

They involve the dependence on the D9-brane Wilson line scalars A^I_i, and are each formally identical to (2.60) (after cancellation of the Klein bottle and 5-brane Möbius diagrams). The relevant values for the coefficients $\mathcal{Q}_{\sigma,\hat{I}}$ can be taken from table 1 and the matrices needed to evaluate the diagrams can be chosen as in (2.51). The only difference compared to the case of $\mathcal{N}=2$ consists of the choice of the matrices W_i that define the Wilson lines. We will come to the explicit calculation of (3.13) in the next section.

The extra diagrams in the $\mathcal{N}=1$ case are

$$\sum_{I=1}^{3}\sum_{J\neq I}\left[\mathcal{K}^{(I)}_{\Theta_J}+\mathcal{M}^{(I)}_{5_J}+\mathcal{A}^{(I)}_{5_J5_J}+\mathcal{A}^{(I)}_{95_J}+\mathcal{A}^{(I)}_{5_J9}\right]+\sum_{\hat{I}=0}^{3}\sum_{J=1}^{3}\sum_{K\neq J}\mathcal{A}^{(\hat{I})}_{5_J5_K}. \tag{3.14}$$

The annulus diagrams with non-trivial insertions among (3.14) immediately vanish due to the properties of the CP matrices. They are proportional to the trace of a traceless matrix $\gamma_{\Theta_I5_J}$.

This leaves us with the diagrams $\mathcal{A}^{(0)}_{5_I5_J}$ and the sum over terms $\mathcal{K}^{(I)}_{\Theta_J}+\mathcal{M}^{(I)}_{5_J}$ in (3.14), which are independent of the Wilson line moduli. Let us first deal with the latter two. From the internal partition function (3.8) and (3.9) it follows that the numerical coefficients that appear are just equal to the coefficients of the amplitudes $\mathcal{K}^{(0)}_{\Theta_I}+\mathcal{K}^{(I)}_{\Theta_I}$ and $\mathcal{M}^{(0)}_{5_I}$, explicitly

$$\mathcal{Q}_{(\Theta_I),J}=\mathcal{Q}_{(\Theta_I),I}=\mathcal{Q}_{(\Theta_I),0}=16\,,\qquad \mathcal{Q}_{(5_I),J}=\mathcal{Q}_{(5_I),0}=-4\,. \tag{3.15}$$

Also, the properties of the orientifold matrices relevant for the Möbius diagrams are identical in both cases, cf. (3.10). The only real difference lies in the fact that the spectrum of KK states that contribute in the traces in $\mathcal{K}^{(I)}_{\Theta_J}+\mathcal{M}^{(I)}_{5_K}$, $I\neq J\neq K$, involves winding states along the untwisted torus $\mathbb{T}^2_J$ (in the open string channel), as opposed to the momentum states that contribute in $\mathcal{K}^{(0)}_{\Theta_J}+\mathcal{K}^{(J)}_{\Theta_J}+\mathcal{M}^{(0)}_{5_J}$. This appears due to the fact that $\mathcal{K}^{(I)}_{\Theta_J}+\mathcal{M}^{(I)}_{5_K}$, $I\neq J\neq K$, is just mapped to $\mathcal{K}^{(0)}_{\Theta_J}+\mathcal{M}^{(0)}_{5_J}$ upon four T-dualities along $\mathbb{T}^4_I$.

To be more precise, a T-duality transformation along $\mathbb{T}^4_I$, i.e. the simultaneous inversion of all four radii of $\mathbb{T}^4_I$, maps, for example, Ω to $\Omega\Theta_I$, thus permuting the insertions in the Klein bottle. It maps D5_I-branes to D9-branes and permutes the other two types of D5-branes, it maps the 9-brane Möbius strip with Ω insertion to the 5_I-brane Möbius with $\Omega\Theta_I$ insertion, and so on.

As observed in (2.67), contributions of the Klein bottle and the 5-brane Möbius strip cancel out according to (both equations hold for each fixed value of I)

$$\begin{aligned}\mathcal{K}^{(0)}_{\Theta_I}+\mathcal{K}^{(I)}_{\Theta_I}+\mathcal{M}^{(0)}_{5_I}&=0\,,\\ \mathcal{M}^{(L)}_{5_K}|_{L\neq I\neq K}+\sum_{J\neq I}\mathcal{K}^{(J)}_{\Theta_I}&=0\,.\end{aligned} \tag{3.16}$$

But note that this leaves us with one of the two diagrams $\mathcal{M}_{5_K}^{(L)}|_{L\neq I\neq K}$ which both have winding modes along $\mathbb{T}_I^2$. Together, this now means that the surviving extra contributions from (3.14) in $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ are given by the following three sets of diagrams,[25]

$$\sum_{J=2}^{3}\sum_{I<J}\left[\left[\mathcal{A}^{(0)}_{5_I5_J}+\mathcal{A}^{(0)}_{5_J5_I}\right]+\mathcal{M}^{(I)}_{5_J}\right]. \tag{3.17}$$

Any set of three is T-dual to $\mathcal{A}^{(0)}_{95_I}+\mathcal{A}^{(0)}_{5_I9}+\mathcal{M}^{(I)}_{9}$ upon four T-dualities along $\mathbb{T}^4_J$. This is precisely the contribution that the $\mathcal{A}_{95}$ and $\mathcal{M}_9$ diagrams give in the $\mathcal{N}=2$ model of section 2.5 (after setting A_i^I to zero), and the result can be obtained from the final result of that section via T-duality. It will be given in section 3.4.

In order to evaluate (3.13) explicitly one needs to use more details of the representation of the CP algebra given in [39], because these amplitudes depend on the Wilson line moduli. We now return to a notation where the D9-branes are broken up into stacks labelled by i, each characterized by its individual value for the three Wilson lines A_i^I. In conventions where the total number of branes is 32, a minimum of 4 branes is required to make up an independent stack that can be associated with a modulus A_i^I. This corresponds to the fact that the maximal D9-brane gauge group of the model is Sp(8) with rank 8, i.e. $4N$ "elementary" D9-branes get identified under the orbifold and orientifold projections to form Sp(N). The D5-branes behave similarly.

3.3 Wilson lines in $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$

The matrix W_i^I for the Wilson line along either one of the two elementary circles of the $\mathbb{T}_I^2$, denoted by the basis vectors e_M^m, $M=1,2$, is used to define the CP matrix

$$\gamma^{(M)}_{W_i^I}=\exp(2\pi i(\vec{a}_i^{\,I}\vec{e}_M)W_i^I)\,. \tag{3.18}$$

Its form is determined by solving a number of constraints [42, 43, 20]. It has to satisfy tadpole cancellation conditions, obey unitarity and be compatible with the orbifold projection. Solutions to these conditions are known explicitly only in a few cases, and we will not go through the procedure in exhaustive detail. We use the definitions of the matrices $\gamma_{\Theta_I i}$ in [39] written in terms of $(4N_i)\times(4N_i)$ matrices,

$$\gamma_{\Theta_1 i}=\begin{bmatrix}0 & -\mathbf{1}_{2N_i}\\ \mathbf{1}_{2N_i} & 0\end{bmatrix}\oplus\mathbf{0}_{32-4N_i}\,,\qquad \gamma_{\Theta_2 i}=\begin{bmatrix}i\sigma_{2N_i} & 0\\ 0 & -i\sigma_{2N_i}\end{bmatrix}\oplus\mathbf{0}_{32-4N_i}\,, \tag{3.19}$$

and $\gamma_{\Theta_3 i}=\gamma_{\Theta_2 i}\gamma_{\Theta_1 i}$, using σ_{2N_i} from (2.52). For concreteness we now discuss the conditions and solutions for a Wilson line along the second torus with $I=2$, but the other cases ($I=1,3$) can be dealt with analogously. Now (skipping the index M for the moment) $\gamma_{W_i^2}$ has to satisfy the tadpole constraints

$$\mathrm{tr}(\gamma_{\Theta_I i}\gamma_{W_i^2})=0\,,\qquad I=1,2,3\,, \tag{3.20}$$

[25]Note that the diagrams (3.17) do not depend on the 9-brane scalars since they do not involve D9-branes at all.

and the compatibility relations

$$(\gamma_{\Theta_1 i}\gamma_{W_i^2})^2 = (\gamma_{\Theta_3 i}\gamma_{W_i^2})^2 = -\mathbf{1}_{4N_i} \oplus \mathbf{0}_{32-4N_i} \,. \tag{3.21}$$

There is no such relation for $\gamma_{\Theta_2 i}$ which is trivial on the second torus by definition. One can now easily convince oneself that any matrix

$$\gamma_{W_i^2} = \mathrm{diag}(e^{-i\varphi}\mathbf{1}_{2N_i}, e^{i\varphi}\mathbf{1}_{2N_i}) \oplus \mathbf{0}_{32-4N_i} \tag{3.22}$$

satisfies these relations and is obviously also unitary on the $(4N_i)\times(4N_i)$ block. In order to make contact to the calculations of the model with $\mathcal{N}=2$ supersymmetry, where we used the diagonal CP matrices (2.51) for $\gamma_{\Theta i}$, we also diagonalize $\gamma_{\Theta_2 i}$ via the unitary transformation

$$P_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & i & 0 & -i \\ 0 & 1 & 0 & 1 \\ -i & 0 & i & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{3.23}$$

such that

$$\begin{aligned} P_4^\dagger \gamma_{\Theta_2 i} P_4 &= \mathrm{diag}(i\mathbf{1}_{2N_i}, -i\mathbf{1}_{2N_i}) \oplus \mathbf{0}_{32-4N_i} \,, \\ P_4^\dagger \gamma^{(M)}_{W_i^2} P_4 &= \mathrm{diag}(e^{i\varphi_M}\mathbf{1}_{N_i}, e^{-i\varphi_M}\mathbf{1}_{N_i}, e^{i\varphi_M}\mathbf{1}_{N_i}, e^{-i\varphi_M}\mathbf{1}_{N_i}) \oplus \mathbf{0}_{32-4N_i} \,. \end{aligned} \tag{3.24}$$

These two matrices are all that is needed to evaluate the traces that occur in the amplitudes with $I=2$ in (3.13). Note that $P_4^\dagger \gamma_{\Theta_2 i} P_4$ is now identical to $\gamma_{\Theta i}$ in (2.51). The continuous real parameters φ_M are interpreted geometrically as the Wilson line degrees of freedom, by identifying them with the projection of the vector $\vec{a}_i^{\,2}$ that appears as the shift in the open string KK momenta onto the two elementary lattice vectors of the second torus $\mathbb{T}_2^2$,

$$\varphi_M = 2\pi\, \vec{e}_M \vec{a}_i^{\,2} \,. \tag{3.25}$$

Using (3.18) this leads to

$$W_i^2 = \mathrm{diag}(\mathbf{1}_{N_i}, -\mathbf{1}_{N_i}, \mathbf{1}_{N_i}, -\mathbf{1}_{N_i}) \oplus \mathbf{0}_{32-4N_i} \,. \tag{3.26}$$

By a unitary change of basis the other Wilson line matrices for $I=1,3$ can also be brought to this form, so that we use the form of (3.26) for all the Wilson lines when evaluating the traces in the next section. The two Wilson lines along the two elementary cycles of the torus are independent, since the orbifold action does not identify the two, and captured by the two independent components of the vector $\vec{a}_i^{\,2}$ in the dual lattice of the torus. The special values $\vec{e}_M \vec{a}_i^{\,2} = \frac{1}{2}$ mod $\mathbb{Z}$ correspond to the positions of the fixed points of the orbifold reflection in the dual lattice. For these values the Wilson lines are of order 2, i.e. $\gamma^2_{W_i^2} = \mathbf{1}_{4N_i} \oplus \mathbf{0}_{32-4N_i}$, and they commute with the projection of the orbifold group since then $[\gamma_{W_i^2}, \gamma_{\Theta_1 i}] = [\gamma_{W_i^2}, \gamma_{\Theta_3 i}] = 0$. These discrete Wilson lines lead to points of enhanced symmetry.

3.4 One-loop Kähler potential

The full set of corrections is now obtained by adding the diagrams (3.17) and (3.13) which we still have to evaluate.

In order to obtain the final expressions for the analogs of the $\mathcal{N}=2$ model (3.13) we use (three copies of) the corrections calculated in section 2.5 and only keep track of the effect of the modified matrices W_i^I (3.26) in place of (2.20) whenever Wilson lines appear. These matrices are not equivalent to the $\mathcal{N}=2$ matrix in (2.20) since they differ by relative signs within the $(2N_i)\times(2N_i)$ blocks. One now has to evaluate the expressions for the 2-point one-loop correlators $\langle V_{T_2^I} V_{T_2^I}\rangle$, which are the analogs of (2.45). Following the same steps as in section 2.5, the correlators are given by a formula as in (2.66) corrected by an additional factor 1/2 for the normalization of the orbifold projection (3.6).

The final result is very similar, except that the trace over CP labels vanishes in the case of the 99 annulus $\mathcal{A}_{ij}$, as follows from the form of (3.26). The evaluation of the traces produces the result

$$\begin{aligned}\sum_\sigma c_\sigma^2 \sum_{k=0,1} \mathrm{tr}\left[E_2(\mathbf{A}_\sigma^I, U^I)\gamma_{\sigma,\hat{I}}\mathcal{Q}_{\sigma,\hat{I}}\right] &= 4\cdot 32\sum_i N_i[E_2(A_i^I,U^I)+E_2(-A_i^I,U^I)] - \\ &\quad -8\sum_i N_i[E_2(2A_i^I,U^I)+E_2(-2A_i^I,U^I)]\,, \end{aligned} \tag{3.27}$$

where the factors changed as compared to (2.67) because now $\sum_i N_i = 8$. We define the quantity

$$\mathcal{E}_2^{\mathbb{Z}_2^2}(A_i^I,U^I) = \sum_\sigma c_\sigma^2 \sum_{k=0,1} \mathrm{tr}\left[E_2(\mathbf{A}_\sigma^I,U^I)\gamma_{\sigma,\hat{I}}\mathcal{Q}_{\sigma,\hat{I}}\right], \tag{3.28}$$

to abbreviate the sum above.

Finally, we have to add the diagrams from (3.17) which we already identified as being T-dual to the contributions above at $A_i^I=0$. Thus, we only have to perform this T-duality on the final result. To do so, note that the function $E_2(A,U)$ that appears in (2.67) and (3.27) is invariant under T-duality at $A=0$, as we show in appendix B. Therefore, the moduli dependence with respect to the U^I appears only through the function $\mathcal{E}_2^{\mathbb{Z}_2^2}(0,U^I)$. The denominator $((S-\bar{S})(T_0^I-\bar{T}_0^I))^{-1}$ from (2.77) transforms under a T-duality along a $\mathbb{T}^4_J$, $J\neq I$, as

$$\frac{1}{(S-\bar{S})(T_0^I-\bar{T}_0^I)} \longrightarrow \frac{1}{(T_0^J-\bar{T}_0^J)(T_0^K-\bar{T}_0^K)}\bigg|_{K\neq I\neq J}. \tag{3.29}$$

Putting the pieces together, the total one-loop correction to the Kähler potential of $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ can be written as (valid up to order $\mathcal{O}(e^{2\Phi})$)

$$K^{(1)}_{\mathbb{Z}_2^2} = \frac{1}{2}\sum_{I=1}^{3}\frac{c\,\mathcal{E}_2^{\mathbb{Z}_2^2}(A_i^I,U^I)}{(S-\bar{S})(T^I-\bar{T}^I)} + \frac{1}{2}\sum_{I=1}^{3}\frac{c\,\mathcal{E}_2^{\mathbb{Z}_2^2}(0,U^I)}{(T^J-\bar{T}^J)(T^K-\bar{T}^K)}\bigg|_{K\neq I\neq J}. \tag{3.30}$$

The constant of proportionality c is the same as for the corrections that appeared in the $\mathcal{N}=2$ model in (2.77) and given in (D.7).

As explained earlier, there is now no reason to expect that one can absorb this term into the logarithm of $K^{(0)}$ as in the case of $\mathcal{N}=2$ supersymmetry, since all higher terms in its expansion would be subject to higher order perturbative corrections.

We will be content with verifying only the component $K_{T^I\bar{T}^I}$ of the Kähler metric and not all other correlators. We expect that everything will go through analogously to the case of $\mathcal{N}=2$ and one can confirm that all 2-point functions are reproduced by the Kähler potential that combines out of (3.3) and (3.30) up to the relevant order in the string coupling.

4. The $\mathcal{N}=1$ orientifold $\mathbb{T}^6/\mathbb{Z}_6'$

The orbifold generator of the orbifold $\mathbb{T}^6/\mathbb{Z}_6'$ model is defined via the vector

$$\vec{v} = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right). \tag{4.1}$$

Since Θ^3 is of order 2, we have D5-branes wrapping the third torus at its fixed points. The maximal gauge group is $(\mathrm{U}(4)^2 \times \mathrm{U}(8))_{\mathrm{D}9} \times (\mathrm{U}(4)^2 \times \mathrm{U}(8))_{\mathrm{D}5}$. The torus lattices of the first and third $\mathbb{T}^2$ must be invariant under Θ, which fixes the complex structures U^1 and U^3, but there are a number of possible choices. This leaves only U^2 as a modulus. In addition, there are three generic untwisted Kähler moduli T^I and S, plus twisted scalars that we do not consider. As for the open string moduli, we consider the Wilson lines A_i^I of the D9-branes, and set the coordinates of the D5-branes and their Wilson lines to zero. We will only present the one-loop correction to the Kähler potential and not discuss the model in full detail, referring to [41] for a thorough discussion of the tree level Kähler potential and coordinates. The form of the untwisted Kähler moduli at leading order is the same as in the case of $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ (up to numerical factors), i.e.

$$\mathrm{Im}(T_0^I) \sim e^{-\Phi}\sqrt{G^I}\,. \tag{4.2}$$

This is all we need to know to write down a vertex operator for T_2^I in analogy with the first line of (2.34). The complexified Wilson line moduli are taken to be the same as above (cf. equation (3.2)) even though only U^2 is still a modulus. (As it turns out, the following calculation does not make use of the exact definition of the Wilson line moduli in this case).

Since we discussed Kähler potential loop corrections at length in previous sections, we can already anticipate the form of the expected correction in this model. This model now contains also $\mathcal{N}=1$ sectors, whose only moduli dependence comes through the vertex operators (2.34) and the Weyl rescaling (2.70). We leave them to future work [44] and here we only focus on the contributions from $\mathcal{N}=2$ sectors. For these contributions, we expect the one-loop correction to the Kähler potential to be of the form

$$K^{(1)}_{\mathbb{Z}_6'} = \frac{1}{3}\,\frac{c_1\mathcal{E}_2^{(2,P)}(A_i^2,U^2)}{(S-\bar{S})(T_0^2-\bar{T}_0^2)} + \frac{1}{3}\,\frac{c_2\mathcal{E}_2^{(2,W)}(0,U^2)}{(T_0^1-\bar{T}_0^1)(T_0^3-\bar{T}_0^3)} + \frac{1}{3}\,\frac{c_3\mathcal{E}_2^{(3)}(A_i^3,U^3)}{(S-\bar{S})(T_0^3-\bar{T}_0^3)} + \cdots\,, \tag{4.3}$$

where the ellipsis stand for the $\mathcal{N}=1$ sectors (we will make a comment about them at the end of this section) and the factors of $\frac{1}{3}$ come from the orbifold projection. The superscripts P and W indicate that the corresponding zero mode sums are from momentum and winding states (in the open string channel), respectively, as will become clearer at the end of the section. As before, in (4.3) we already set to zero the 5-brane scalars, that would otherwise have appeared in the first argument of $\mathcal{E}_2^{(2,W)}(0,U^2)$. Let us now further restrict attention to Wilson lines A_i^3 along the third two-torus, that we will denote simply as A_i in this section. Here we determine the form of $\mathcal{E}_2^{(2,P)}(0,U^2)$, $\mathcal{E}_2^{(2,W)}(0,U^2)$ and $\mathcal{E}_2^{(3)}(A_i,U^3)$, but leave the constants c_i and the $\mathcal{N}=1$ sectors for the future [44].

We will only explain a few ingredients of this computation, following the steps of the previous sections. The mother of all inventions is indolence, so we would like to find a means of reducing calculations in this model to the K3$\times\mathbb{T}^2$ case as much as possible.To do so we first have to introduce some new notation. Define an "untwisted torus twist-vector component" $v_{\rm untw}=v_2$ for $k=2,4$, $v_{\rm untw}=v_3$ for $k=3$, and similarly $v_{\rm tw}=v_3$ for $k=2,4$, $v_{\rm tw}=v_2$ for $k=3$. Then, since $kv_{\rm untw}$ is integer,

$$\frac{\vartheta\left[{\alpha \atop \beta+kv_{\rm untw}}\right]}{\eta^3}\frac{\vartheta\left[{\alpha+h \atop \beta+kv_1}\right]\vartheta\left[{\alpha+h \atop \beta+kv_{\rm tw}}\right]}{\vartheta\left[{1/2+h \atop 1/2+kv_1}\right]\vartheta\left[{1/2+h \atop 1/2+kv_{\rm tw}}\right]}=(-1)^{kv_{\rm untw}}\frac{\vartheta\left[{\alpha \atop \beta}\right]}{\eta^3}\frac{\vartheta\left[{\alpha+h \atop \beta+kv_1}\right]\vartheta\left[{\alpha-h \atop \beta-kv_1}\right]}{\vartheta\left[{1/2+h \atop 1/2+kv_1}\right]\vartheta\left[{1/2-h \atop 1/2-kv_1}\right]} \tag{4.4}$$

where we used $kv_{\rm tw}=-k(v_1+v_{\rm untw})$. The phase on the right hand side is independent of the spin structure. At most it gives an overall sign

$$(-1)^{kv_{\rm untw}}=\begin{cases}-1 & k=2,3\\ \;\;1 & k=4\end{cases}. \tag{4.5}$$

The internal partition function in $\mathcal{N}=2$ sectors is then reduced to K3$\times\mathbb{T}^2$ form up to overall factors

$$\begin{aligned}\mathcal{Z}^{\rm int}_{\sigma,k}\left[{\alpha \atop \beta}\right]&=\eta_{\alpha\beta}\frac{\vartheta\left[{\alpha \atop \beta}\right](0,\tau)}{\eta^3(\tau)}\frac{\vartheta\left[{\alpha+h \atop \beta+g}\right](0,\tau)\vartheta\left[{\alpha-h \atop \beta-g}\right](0,\tau)}{\vartheta\left[{1/2+h \atop 1/2+g}\right](0,\tau)\vartheta\left[{1/2-h \atop 1/2-g}\right](0,\tau)}\times\\ &\times(-1)^{kv_{\rm untw}}\times\begin{cases}a_k & \text{for } h=0\\ 1 & \text{for } h=\frac{1}{2}\end{cases},\end{aligned} \tag{4.6}$$

where h is as before, while $g=kv_1$ whenever there is a Θ acting on the world-sheet oscillators in the trace. The trigonometric factors a_k can be read off from appendix A.1 and have been evaluated to

$$a_k=\begin{cases}3 & k=2\\ 4 & k=3\\ -3 & k=4\end{cases}. \tag{4.7}$$

To perform the sum over spin structures we deal with the first two and the third terms in the Kähler potential (4.3) separately. Let us confront the more involved third term first.

	Sector	k	$\mathcal{Q}_{\sigma,k}$	$\gamma_{\sigma,k}$
$\mathcal{K}$	$\sigma = (1)$	$0,1,2,3,4,5$	0	
	$\sigma = (\Theta^3)$	$0,3^{\spadesuit}$	16	
$\mathcal{A}_{99}$	$\sigma = (ij)$	0	0	$\gamma_i \otimes \gamma_j^{-1}$
	$\sigma = (ij)$	$2,3,4$	$0,-4,0$	$\gamma_{\Theta i}^k \otimes \gamma_{\Theta j}^{-k}$
$\mathcal{A}_{55}$	$\sigma = (ab)$	0	0	$\gamma_a \otimes \gamma_b^{-1}$
	$\sigma = (ab)$	$2,3,4$	$0,-4,0$	$\gamma_{\Theta a}^k \otimes \gamma_{\Theta b}^{-k}$
$\mathcal{A}_{95}$	$\sigma = (ia)$	0	1	$\gamma_i \otimes \gamma_a^{-1}$
	$\sigma = (ia)$	$3^{\spadesuit}$	-1	$\gamma_{\Theta i}^k \otimes \gamma_{\Theta a}^{-k}$
$\mathcal{M}_9$	$\sigma = (i)$	0	0	$-\gamma_{\Omega i}^T \gamma_{\Omega i}^{-1}$
	$\sigma = (i)$	$2,3,4$	$0,-4,0$	$-\gamma_{\Omega\Theta i}^{Tk} \gamma_{\Omega\Theta i}^{-k}$
$\mathcal{M}_5$	$\sigma = (a)$	0	-4	$-\gamma_{\Omega a}^T \gamma_{\Omega a}^{-1}$
	$\sigma = (a)$	$2,3,4$	$0,0,0$	$-\gamma_{\Omega\Theta a}^{Tk} \gamma_{\Omega\Theta a}^{-k}$

Table 2: $\mathcal{N}=2$ sector amplitudes in $\mathbb{Z}_6'$ for $\langle V_{T_2^3} V_{T_2^3} \rangle$. The $\spadesuit$ remind us that $\mathcal{A}_{95}^{(k=2,4)}, \mathcal{K}_{\Theta^3}^{(k=2,4)}$ are $\mathcal{N}=1$, so they are not included.

The vertex operators in $\langle V_{T_2^3} V_{T_2^3} \rangle$ are polarized along the third torus. The moduli-dependent part of the third term in (4.3) then only receives contributions from the insertions $k=0,3$. For example, using (A.17) we see that for $\mathcal{A}_{99}^{(3)}$, $\mathcal{Q}_{\sigma,k} = a_k(-1)^{kv_{\mathrm{untw}}} = -4$. All results we need are listed in table 2, in particular all the $\mathcal{N}=2$, $k=2,4$ sector amplitudes vanish (for the 2-point function of $V_{T_2^3}$; this will be different for the 2-point function of $V_{T_2^2}$ discussed in a moment).

For the following details on the Chan-Paton factors we refer to [37, 20]. The action of Θ is

$$\gamma_{\Theta i} = \mathrm{diag}\Big(\beta \mathbf{1}_{4-N_i}, \beta^5 \mathbf{1}_{4-N_i}, \beta^9 \mathbf{1}_{8-N_i}, \bar\beta \mathbf{1}_{4-N_i}, \bar\beta^5 \mathbf{1}_{4-N_i}, \bar\beta^9 \mathbf{1}_{8-N_i}, \gamma_{\Theta i}^{[6N_i]}\Big) \tag{4.8}$$

with

$$\gamma_{\Theta i}^{[6N_i]} = \mathrm{diag}(\beta, \beta^5, \beta^9, \bar\beta, \bar\beta^5, \bar\beta^9) \otimes \mathbf{1}_{N_i} = \gamma_{\Theta i}^{[6]} \otimes \mathbf{1}_{N_i} \tag{4.9}$$

and $\beta = e^{i\pi/6}$. The remaining gauge group is

$$\mathrm{U}(4-N_i)^2 \times \mathrm{U}(8-N_i) \times \mathrm{U}(N_i)\,, \tag{4.10}$$

and tadpole cancellation requires $\sum_i N_i = 4$. In the $k=3$ sector, the action of the orbifold twist on the Chan-Paton factors is particularly simple

$$(\gamma_{\Theta i}^{[6]})^3 = \mathrm{diag}(i\mathbf{1}_3, -i\mathbf{1}_3)\,. \tag{4.11}$$

There is a convenient basis where the Wilson line on the D9-branes is diagonal,

$$\begin{aligned} \gamma_{W_i} &= \mathrm{diag}(\mathbf{1}_{32-6N_i}, \gamma_{W_i}^{[6N_i]})\,, \\ \gamma_{W_i}^{[6N_i]} &= \mathrm{diag}(e^{i\vec e\,\vec a}, e^{i\vec e\,\vec a^{\Theta}}, e^{i\vec e\,\vec a^{\Theta^2}}, e^{-i\vec e\,\vec a}, e^{-i\vec e\,\vec a^{\Theta}}, e^{-i\vec e\,\vec a^{\Theta^2}}) \otimes \mathbf{1}_{N_i}\,. \end{aligned} \tag{4.12}$$

This is very similar to the matrix (2.20) but one has to sum over images of Θ in addition. Just like in the $\mathrm{K3}\times\mathbb{T}^2$ case, all other matrices with $\mathcal{Q}_{\sigma,k}\neq 0$ are proportional to identity matrices. Then the only non-vanishing moduli-dependent contributions come from diagrams

$$\mathcal{K}^{(0)}_{\Theta^3}+\mathcal{K}^{(3)}_{\Theta^3}+\mathcal{A}^{(3)}_{99}+\mathcal{A}^{(0)}_{95}+\mathcal{A}^{(0)}_{59}+\mathcal{M}^{(3)}_{9}+\mathcal{M}^{(0)}_{5}\,. \tag{4.13}$$

The only dependence on the Wilson lines along the third two-torus comes from $\mathcal{N}=2$ sectors

$$\mathcal{A}^{(3)}_{99}+\mathcal{A}^{(0)}_{95}+\mathcal{A}^{(0)}_{59}+\mathcal{M}^{(3)}_{9}\,. \tag{4.14}$$

Now we can follow the steps in section 2, in particular going from (2.60) to (2.67). This leads to

$$\begin{aligned}
&\sum_\sigma c_\sigma^2\sum_{k=0,3}\mathrm{tr}\left[E_2(\mathbf{A}_\sigma,U^3)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right]=\\
&\quad=4E_2(0,U^3)\left[\mathcal{Q}_{(\Theta^3),0}+\mathcal{Q}_{(\Theta^3),3}\right]+\sum_{i,j}\mathrm{tr}\left[E_2(\mathbf{A}_{(ij)},U^3)\gamma_{(ij),3}\mathcal{Q}_{(ij),3}\right]+\\
&\qquad+2\sum_{i,a}\mathrm{tr}\left[E_2(\mathbf{A}_{(i)},U^3)\gamma_{(ia),0}\mathcal{Q}_{(ia),0}\right]+\sum_{i}\mathrm{tr}\left[E_2(\mathbf{A}_{(i)},U^3)\gamma_{(i),3}\mathcal{Q}_{(i),3}\right]+\\
&\qquad+\sum_{a}\mathrm{tr}\left[E_2(0,U^3)\gamma_{(a),0}\mathcal{Q}_{(a),0}\right]\\
&\quad=\sum_{m=0}^{2}\Bigg\{\sum_{n=0}^{2}\sum_{i,j}\Big[-4N_iN_j\Big[E_2(A_i^{\Theta_m}-A_j^{\Theta_n},U^3)+E_2(-A_i^{\Theta_m}+A_j^{\Theta_n},U^3)-\\
&\qquad\qquad-E_2(A_i^{\Theta_m}+A_j^{\Theta_n},U^3)-E_2(-A_i^{\Theta_m}-A_j^{\Theta_n},U^3)\Big]\Big]+\\
&\qquad\quad+2\cdot 32\sum_i N_i\Big[E_2(A_i^{\Theta_m},U^3)+E_2(-A_i^{\Theta_m},U^3)\Big]-\\
&\qquad\quad-4\sum_i N_i\Big[E_2(2A_i^{\Theta_m},U^3)+E_2(-2A_i^{\Theta_m},U^3)\Big]\Bigg\}\,.
\end{aligned}\tag{4.15}$$

We denote this quantity as $\mathcal{E}_2^{(3)}(A_i,U^3)$ as introduced in (4.3). As emphasized above, this complex structure U^3 is no longer a modulus, since it is fixed by the action of the orbifold.

The functions $\mathcal{E}_2^{(2,P)}(0,U^2)$ and $\mathcal{E}_2^{(2,W)}(0,U^2)$ in (4.3) can be determined by considering vertex operators polarized along the second torus in $\langle V_{T_2^2}V_{T_2^2}\rangle$. Part of the calculation was already done in [14] since we have not turned on Wilson lines along this two-torus. The function $\mathcal{E}_2^{(2,P)}(0,U^2)$ gets contributions from

$$\mathcal{K}^{(2,4)}_{1}+\mathcal{A}^{(2,4)}_{99}+\mathcal{M}^{(2,4)}_{9}\,, \tag{4.16}$$

whereas $\mathcal{E}_2^{(2,W)}(0,U^2)$ gets contributions from

$$\mathcal{K}^{(1,5)}_{1}+\mathcal{A}^{(2,4)}_{55}+\mathcal{M}^{(2,4)}_{5}\,. \tag{4.17}$$

Note that the diagrams in (4.16) involve a momentum sum along the second torus, while the diagrams in (4.17) involve a sum over winding states along the second torus (both in the open channel). For the Klein bottle this is clear from the appendix of [37]. We do not go through the details of the calculation again. As in section (2.5), the diagrams with the momentum sum lead to a term in the Kähler potential proportional to

$$\frac{E_2(0,U^2)}{(S-\bar{S})(T^2-\bar{T}^2)}\,, \tag{4.18}$$

whereas the different volume dependence in the amplitudes with winding sums change the volume dependent factor to[26]

$$\frac{E_2(0,U^2)}{(T^1-\bar{T}^1)(T^3-\bar{T}^3)}\,. \tag{4.19}$$

This is analogous to the second term in (3.30), which also comes from terms involving winding sums as opposed to momentum sums. Thus, both $\mathcal{E}_2^{(2,P)}(0,U^2)$ and $\mathcal{E}_2^{(2,W)}(0,U^2)$ are proportional to $E_2(0,U^2)$ and the factor of proportionality is given by a trace similar to (4.15). Since we did not determine the constants c_1, c_2, c_3 in (4.3), we do not give the proportionality factor here, but we hope to come back to a more complete study of this model in the future [44].

Let us just make one final comment on the $\mathcal{N}=1$ sectors. One can infer from the 2-point functions of the T^I, for instance, that there may be contributions to the Kähler potential proportional to

$$\frac{C}{\sqrt{(S-\bar{S})(T^1-\bar{T}^1)(T^2-\bar{T}^2)(T^3-\bar{T}^3)}}\,, \tag{4.20}$$

for some constant C. This can be seen as follows. Let us choose the 2-point function of T^1 to be concrete (the other cases $I=2,3$ are analogous). Its moduli dependence comes only from the vertex operators (giving a factor $(T^1-\bar{T}^1)^{-2}$) and the Weyl rescaling (cf. (2.70)), together giving a factor

$$\frac{1}{(T^1-\bar{T}^1)^2}\frac{e^{\Phi}}{S-\bar{S}} \sim \frac{1}{\sqrt{(S-\bar{S})(T^1-\bar{T}^1)^5(T^2-\bar{T}^2)(T^3-\bar{T}^3)}}\,. \tag{4.21}$$

Integrating this twice with respect to T^1 and $\bar{T}^1$ would lead to a scaling behavior as in (4.20). However, if the $\mathcal{N}=1$ sectors only produce IR-divergent terms (cf. [14]), these would not be included in the Kähler potential.

5. Conclusions

We would like to conclude with two comments. The first concerns the translation of our results to the language of D3- and D7-branes, which is more useful for most phenomenological applications. To do so, one performs six T-dualities along all internal circles. This maps D9- to D3-branes, and it maps D5-branes wrapping some $\mathbb{T}^2$ onto D7-branes on the transverse $\mathbb{T}^4$. Further, world-sheet parity Ω maps to $\Omega(-1)^6(-1)^{F_R}$, where $(-1)^6$ is the

[26]The dependence on U^2 does not change as can be seen again from the $\mathrm{SL}(2,\mathbb{Z})$-invariance of $E_2(0,U^2)$, cf. appendix B.

reflection along all six circles and F_R the right-moving space-time fermion number. This operation is obviously not a symmetry of the models we studied in this paper, but rather maps an orientifold of one formulation into another orientifold of the other formulation, the two being physically identical at the orbifold point.

Thus we can just copy the results of this paper, i.e. (2.77) for $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$, (3.30) for $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ and (4.3) for $\mathbb{T}^6/\mathbb{Z}_6'$, if we take into account that they now depend on the T-dual variables.[27] For the $\mathcal{N}=2$ model of section 2 these are given by the T-dual complex structure modulus, the positions of the D3-branes (cf. (B.8) and (B.18), respectively, for $A=D=0$ and $B=-C=1$) and

$$\begin{aligned} S &\mapsto \frac{1}{\sqrt{8\pi^2}}(C_0 + ie^{-\Phi})\,, \\ S' &\mapsto \frac{1}{\sqrt{8\pi^2}}(C_4|_{\mathbb{T}^4} + ie^{-\Phi}\mathcal{V}_{\mathrm{K3}}) + \frac{1}{8\pi}\sum_i N_i(U(a^5)_i + (a^4)_i(a^5)_i)\,. \end{aligned} \tag{5.1}$$

Thus, whereas S becomes independent of the volume, the leading term in the imaginary part of S' becomes the volume of a 4-cycle, as measured in the ten-dimensional Einstein frame metric, $e^{-\Phi}\mathcal{V}_{\mathrm{K3}}^{(\mathrm{string})} = \mathcal{V}_{\mathrm{K3}}^{(\mathrm{Einstein})}$. The mapping in the case of $\mathcal{N}=1$ orientifolds is analogous. This implies, for instance, that the first correction term in (3.30) scales with the total Einstein-frame volume $\mathcal{V}^{(\mathrm{Einstein})}$ like

$$\frac{1}{(S-\bar{S})(T_0^I - \bar{T}_0^I)} \sim e^{\Phi}[\mathcal{V}^{(\mathrm{Einstein})}]^{-\frac{2}{3}}\,, \tag{5.2}$$

where we assumed that none of the three 4-cycles degenerates. Compared to the tree level α' corrections that were determined in [5], this term dominates at large volume.[28] We will pursue this issue further in our companion paper [2].

Moreover, in the T-dual coordinates, the corrections depend on the scalars parametrizing the 3-brane positions and should be relevant for brane inflation models based on mobile D3-branes.

Finally, we would like to remark that a different (closed string) approach to corrections to Kähler potentials in (warped) string compactifications has been pursued in [46]. It would be interesting to explore if their results are related to ours, or if they capture a complementary type of corrections that one would have to take into account in addition to ours, in applications to more realistic warped compactifications.

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[27] The form invariance of $E_2(A,U)$ under $\mathrm{SL}(2,\mathbb{Z})$-transformations is demonstrated in appendix B.

[28] A similar scaling of the one-loop correction to the Kähler potential in models with D3/D7-branes was reported in [45].

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A. Amplitude toolbox

A.1 Partition functions

The zero modes and oscillators of the external space time coordinates and ghosts give the same contribution for all compactifications considered in this paper:

$$\mathcal{Z}_4\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau)=\frac{1}{4\pi^4\tau^2}\frac{\vartheta[{\alpha\atop\beta}](0,\tau)}{\eta(\tau)^3}\,. \tag{A.1}$$

For the $\mathbb{T}^6/\mathbb{Z}_6'$ model, the internal D9-D9 annulus partition functions are:

$$\begin{aligned}
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\prod_{j=1}^{3}(-2\sin(\pi k v_j))\,\frac{\vartheta[{\alpha\atop\beta+kv_j}](0,\tau)}{\vartheta[{1/2\atop 1/2+kv_j}](0,\tau)}\,, \qquad k=1,5\,,\\
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\,\frac{\vartheta[{\alpha\atop\beta+kv_2}](0,\tau)}{\eta(\tau)^3}\prod_{j=1,3}(2\sin(\pi k v_j))\frac{\vartheta[{\alpha\atop\beta+kv_j}](0,\tau)}{\vartheta[{1/2\atop 1/2+kv_j}](0,\tau)}\,, \qquad k=2,4\,,\\
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\,\frac{\vartheta[{\alpha\atop\beta+3v_3}](0,\tau)}{\eta(\tau)^3}\prod_{j=1,2}(2\sin(3\pi v_j))\,\frac{\vartheta[{\alpha\atop\beta+3v_j}](0,\tau)}{\vartheta[{1/2\atop 1/2+3v_j}](0,\tau)}\,, \qquad k=3\,.
\end{aligned}\tag{A.2}$$

The internal D9-D5 annulus partition functions are:

$$\begin{aligned}
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\,(-2\sin(\pi k v_3))\,\frac{\vartheta[{\alpha\atop\beta+3v_3}](0,\tau)}{\vartheta[{1/2\atop 1/2+3v_3}](0,\tau)}\prod_{j=1}^{2}\frac{\vartheta[{\alpha+1/2\atop\beta+kv_j}](0,\tau)}{\vartheta[{1/2+1/2\atop 1/2+kv_j}](0,\tau)}\,, \qquad k=1,2,4,5\,,\\
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\,\frac{\vartheta[{\alpha\atop\beta+kv_3}](0,\tau)}{\eta(\tau)^3}\prod_{j=1,2}\frac{\vartheta[{\alpha+1/2\atop\beta+kv_j}](0,\tau)}{\vartheta[{1/2+1/2\atop 1/2+kv_j}](0,\tau)}\,, \qquad k=0,3\,.
\end{aligned}\tag{A.3}$$

The internal D9 Möbius strip partition functions are:

$$\begin{aligned}
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\prod_{j=1}^{3}(-2\sin(\pi k v_j))\,\frac{\vartheta[{\alpha\atop\beta+kv_j}](0,\tau)}{\vartheta[{1/2\atop 1/2+kv_j}](0,\tau)}\,, \qquad k=1,5\,,\\
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\,\frac{\vartheta[{\alpha\atop\beta+kv_2}](0,\tau)}{\eta(\tau)^3}\prod_{j=1,3}(2\sin(\pi k v_j))\frac{\vartheta[{\alpha\atop\beta+kv_j}](0,\tau)}{\vartheta[{1/2\atop 1/2+kv_j}](0,\tau)}\,, \qquad k=2,4\,,\\
\mathcal{Z}_k^{\rm int}\begin{bmatrix}\alpha\\\beta\end{bmatrix}(\tau) &= \eta_{\alpha\beta}\,\frac{\vartheta[{\alpha\atop\beta+3v_3}](0,\tau)}{\eta(\tau)^3}\prod_{j=1,2}(2\sin(3\pi v_j))\,\frac{\vartheta[{\alpha\atop\beta+3v_j}](0,\tau)}{\vartheta[{1/2\atop 1/2+3v_j}](0,\tau)}\,, \qquad k=3\,.
\end{aligned}\tag{A.4}$$

The arguments are $\tau = 1/2 + it/2$ for the Möbius strip and $\tau = it$ for the annulus (see figure 3). The above amplitudes can all be read off from appendix B of [47]. For the internal partition functions of the D5-D5 annulus and the D5 Möbius strip we refer the reader to the appendix of [37], where also the Klein bottle partition functions are given as

$$\begin{aligned}
\mathcal{Z}^{\rm int}_{(1),k}\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] &= \prod_{i=1}^{3} \frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+2kv_i\end{smallmatrix}\right](0,\tau)(-2\sin(2\pi k v_i))}{\vartheta\left[\begin{smallmatrix}1/2\\ 1/2+2kv_i\end{smallmatrix}\right](0,\tau)}\,, \qquad\qquad (A.5)\\
\mathcal{Z}^{\rm int}_{(\Theta^3),k}\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] &= \tilde{\chi}(\Theta^3,\Theta^k)\left(\prod_{i=1}^{2} \frac{\vartheta\left[\begin{smallmatrix}\alpha+1/2\\ \beta+2kv_i\end{smallmatrix}\right](0,\tau)}{\vartheta\left[\begin{smallmatrix}0\\ 1/2+2kv_i\end{smallmatrix}\right](0,\tau)}\right) \frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+2kv_3\end{smallmatrix}\right](0,\tau)(-2\sin(2\pi k v_3))}{\vartheta\left[\begin{smallmatrix}1/2\\ 1/2+2kv_3\end{smallmatrix}\right](0,\tau)}\,,
\end{aligned}$$

where the second argument of the theta functions is $\tau = 2it$, as usual for the Klein bottle, and the number of simultaneous fixed points of Θ^3 and Θ^k is

$$\tilde{\chi}(\Theta^3,\Theta^k) = \begin{cases} 12 & k = 1,5 \\ 12 & k = 2,4 \\ 16 & k = 3 \end{cases}. \qquad (A.6)$$

A.2 World-sheet correlators

The bosonic correlation function on the torus $\mathcal{T}$ in the untwisted directions is

$$\langle X(\nu_1)X(\nu_2)\rangle_{\mathcal{T}} = -\frac{\alpha'}{2}\ln\left|\frac{2\pi}{\vartheta_1'(0,\tau)}\vartheta_1\left(\frac{\nu_1-\nu_2}{2\pi},\tau\right)\right|^2 + \alpha'\frac{(\mathrm{Im}(\nu_1-\nu_2))^2}{4\pi\,\mathrm{Im}(\tau)}\,. \qquad (A.7)$$

The correlators on the annulus $\mathcal{A}$, Möbius strip $\mathcal{M}$ and Klein bottle $\mathcal{K}$ are obtained by symmetrizing this function under the involutions

$$I_{\mathcal{A}}(\nu) = I_{\mathcal{M}}(\nu) = 2\pi - \bar{\nu}\,, \qquad I_{\mathcal{K}}(\nu) = 2\pi - \bar{\nu} + \pi\tau \qquad (A.8)$$

producing (cf. the appendix of [10])

$$\langle X(\nu_1)X(\nu_2)\rangle_\sigma = \langle X(\nu_1)X(\nu_2)\rangle_\sigma + \langle X(\nu_1)X(I(\nu_2))\rangle_\sigma\,, \qquad (A.9)$$

where $\sigma \in \{\mathcal{A},\mathcal{M},\mathcal{K}\}$. For untwisted world-sheet fermions in the even spin structures, the correlation functions on the torus and with DN boundary conditions are, respectively,

$$\begin{aligned}
P_F(s,\nu_1,\nu_2)\delta^{\mu\nu} \equiv \langle\psi^\mu(\nu_1)\psi^\nu(\nu_2)\rangle^{\alpha,\beta}_{\mathcal{T}} &= \frac{1}{2\pi}\frac{\vartheta[^\alpha_\beta](\frac{\nu_1-\nu_2}{2\pi},\tau)\vartheta_1'(0,\tau)}{\vartheta[^\alpha_\beta](0,\tau)\vartheta_1(\frac{\nu_1-\nu_2}{2\pi},\tau)}\delta^{\mu\nu}\\
&= \overline{\langle\tilde{\psi}^\mu(\bar{\nu}_1)\tilde{\psi}^\nu(\bar{\nu}_2)\rangle^{\alpha,\beta}_{\mathcal{T}}}\,, \qquad (A.10)\\
\langle\psi^m(\nu_1)\psi^n(\nu_2)\rangle^{\alpha,\beta}_{\mathcal{A}_{95}} &= \frac{1}{2\pi}\frac{\vartheta[^{\alpha+1/2}_{\beta}](\frac{\nu_1-\nu_2}{2\pi},\tau)\vartheta_1'(0,\tau)}{\vartheta[^{\alpha+1/2}_{\beta}](0,\tau)\vartheta_1(\frac{\nu_1-\nu_2}{2\pi},\tau)}G^{mn}\\
&= \overline{\langle\tilde{\psi}^m(\bar{\nu}_1)\tilde{\psi}^n(\bar{\nu}_2)\rangle^{\alpha,\beta}_{\mathcal{A}_{95}}}\,, \qquad (A.11)
\end{aligned}$$

$\left[\begin{smallmatrix}\alpha\\\beta\end{smallmatrix}\right]$	s	$\eta_{\alpha\beta}$	$(-1)^{s+1}$	$\tau\to-1/\tau$
$\left[\begin{smallmatrix}1/2\\1/2\end{smallmatrix}\right]$	1	-1	1	$\to -\left[\begin{smallmatrix}1/2\\1/2\end{smallmatrix}\right]$
$\left[\begin{smallmatrix}1/2\\0\end{smallmatrix}\right]$	2	-1	-1	$\to \left[\begin{smallmatrix}0\\1/2\end{smallmatrix}\right]$
$\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]$	3	1	1	$\to \left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]$
$\left[\begin{smallmatrix}0\\1/2\end{smallmatrix}\right]$	4	-1	-1	$\to \left[\begin{smallmatrix}1/2\\0\end{smallmatrix}\right]$

Table 3: Some reminders for the summation over spin structures. The last column denotes the transformation of the partition function $\mathcal{Z}[^{\alpha}_{\beta}]$, so after modular transformation, $\eta_{\alpha\beta}$ is equivalent to $(-1)^{1+s}$, which is sometimes used in the literature.

where the relation between α, β and s is listed in table 3. The propagators for twisted world-sheet fermions in the even spin structures on the torus and with ND boundary conditions are, respectively,

$$\langle\psi^m(\nu_1)\psi^n(\nu_2)\rangle_{\mathcal{T}}^{\alpha,\beta} = \frac{1}{2\pi}\frac{\vartheta[^{\alpha}_{\beta+kv_i}](\frac{\nu_1-\nu_2}{2\pi},\tau)\vartheta_1'(0,\tau)}{\vartheta[^{\alpha}_{\beta+kv_i}](0,\tau)\vartheta_1(\frac{\nu_1-\nu_2}{2\pi},\tau)}G^{mn}\,, \tag{A.12}$$

$$\langle\psi^m(\nu_1)\psi^n(\nu_2)\rangle_{\mathcal{A}_{95}}^{\alpha,\beta} = \frac{1}{2\pi}\frac{\vartheta[^{\alpha+1/2}_{\beta+kv_i}](\frac{\nu_1-\nu_2}{2\pi},\tau)\vartheta_1'(0,\tau)}{\vartheta[^{\alpha+1/2}_{\beta+kv_i}](0,\tau)\vartheta_1(\frac{\nu_1-\nu_2}{2\pi},\tau)}G^{mn}\,. \tag{A.13}$$

Just as for bosons, fermion propagators for the remaining surfaces can be determined from the torus propagators by the method of images. The result was listed in the appendix of [10], but as we are using slightly different conventions, we summarize the derivation in section A.4.

$$\begin{aligned}\langle\psi(\nu_1)\psi(\nu_2)\rangle_{\sigma}^{\alpha,\beta} &= P_F(s,\nu_1,\nu_2)\,, \qquad \sigma\in\{\mathcal{A},\mathcal{M},\mathcal{K}\}\\ \langle\psi(\nu_1)\tilde{\psi}(\bar{\nu}_2)\rangle_{\sigma}^{\alpha,\beta} &= iP_F(s,\nu_1,I_\sigma(\nu_2))\,,\\ \langle\tilde{\psi}(\bar{\nu}_1)\tilde{\psi}(\bar{\nu}_2)\rangle_{\sigma}^{\alpha,\beta} &= \overline{P_F(\bar{s},\nu_1,\nu_2)}\,,\end{aligned} \tag{A.14}$$

where $P_F(s,\nu_1,\nu_2)$ was defined in (A.10).

A.3 Mathematical identities

The fermionization identity in [32] (13.4.20)–(13.4.21) when at least one of the ϕ_i' is zero can be reorganized as

$$\sum_{\substack{\alpha\beta\\ \text{even}}}\eta_{\alpha\beta}\prod_{i=1}^{4}\vartheta\left[\begin{smallmatrix}\alpha\\\beta\end{smallmatrix}\right](\phi_i,\tau) = \prod_{i=1}^{4}\vartheta\left[\begin{smallmatrix}1/2\\1/2\end{smallmatrix}\right](\phi_i,\tau)\,,$$

which greatly simplifies the integrands for even spin structures (here "even" means all except $(1/2,1/2)$). Note that the "angles" ϕ_i are equivalent to shifts in the characteristic

β. Using periodicity properties one can generalize this formula to allow for shifts also in the α characteristic:

$$\sum_{\substack{\alpha\beta \\ \text{even}}} \eta_{\alpha\beta}\, \vartheta\Big[{\alpha \atop \beta}\Big](\nu,\tau)\vartheta\Big[{\alpha+h_1 \atop \beta+g_1}\Big](\nu,\tau)\prod_{i=2}^{3}\vartheta\Big[{\alpha+h_i \atop \beta+g_i}\Big](0,\tau) =$$

$$= \vartheta\Big[{1/2 \atop 1/2}\Big](\nu,\tau)\vartheta\Big[{1/2+h_1 \atop 1/2+g_1}\Big](\nu,\tau)\prod_{i=2}^{3}\vartheta\Big[{1/2+h_i \atop 1/2+g_i}\Big](0,\tau) \tag{A.15}$$

where $\sum g_i = \sum h_i = 0$. We will also need the periodicity formulas

$$\vartheta\Big[{\alpha \atop \beta}\Big](\nu \pm \tau,\tau) = e^{-i\pi\tau\mp 2i\pi(\nu+\beta)}\vartheta\Big[{\alpha \atop \beta}\Big](\nu,\tau)\,, \tag{A.16}$$

that can easily be derived from the sum representations for the theta functions in [32], as is

$$\vartheta\Big[{\alpha \atop \beta+n}\Big](\nu,\tau) = (-1)^{2\alpha n}\vartheta\Big[{\alpha \atop \beta}\Big](\nu,\tau) \tag{A.17}$$

for integer n. Finally, we often make use of the Poisson resummation formula

$$\vartheta\Big[{\vec{\alpha} \atop \vec{0}}\Big](0,itG^{-1}) = \sqrt{G}\,t^{-N/2}\,\vartheta\Big[{\vec{0} \atop \vec{0}}\Big](\vec{\alpha},it^{-1}G)\,, \tag{A.18}$$

for

$$\vartheta\Big[{\vec{\alpha} \atop \vec{\beta}}\Big](\vec{\nu},G) = \sum_{\vec{n}\in\mathbb{Z}^N} e^{i\pi(\vec{n}+\vec{\alpha})^{\mathrm{T}}G(\vec{n}+\vec{\alpha})}e^{2\pi i(\vec{\nu}+\vec{\beta})^{\mathrm{T}}(\vec{n}+\vec{\alpha})}\,, \tag{A.19}$$

where G is an $N\times N$ matrix with $\mathrm{Im}(G) > 0$.

A.4 Method of images for fermions

In this subsection we give some more details about the fermion correlators in (A.14). Throughout, we will use the conventions of [32], in particular $z = e^{-i\nu}$. Let us begin by checking that the correlators given in (A.10) and (A.12) have short-distance expansions consistent with their OPEs. To see this, we take the same Laurent expansion in z as [32], i.e.

$$\psi^\mu(z) = \sum_{r\in\mathbb{Z}+\gamma}\frac{\psi^\mu_r}{z^{r+1/2}}\,, \qquad \tilde{\psi}^\mu(\bar{z}) = \sum_{r\in\mathbb{Z}+\tilde{\gamma}}\frac{\tilde{\psi}^\mu_r}{\bar{z}^{r+1/2}}\,, \tag{A.20}$$

where γ and $\tilde{\gamma} = 0$ (1/2) in the R (NS) sector. We also take the same OPEs, i.e.

$$\psi^\mu(z_1)\psi^\nu(z_2) \sim \frac{\eta^{\mu\nu}}{z_1 - z_2}\,, \qquad \tilde{\psi}^\mu(\bar{z}_1)\tilde{\psi}^\nu(\bar{z}_2) \sim \frac{\eta^{\mu\nu}}{\bar{z}_1 - \bar{z}_2}\,. \tag{A.21}$$

Using the conformal transformation to torus coordinates

$$\begin{aligned} \psi^\mu(z) &= (\partial_z\nu)^{1/2}\psi^\mu(\nu) = (-iz)^{-1/2}\psi^\mu(\nu)\,, \\ \tilde{\psi}^\mu(\bar{z}) &= (\partial_{\bar{z}}\bar{\nu})^{1/2}\tilde{\psi}^\mu(\bar{\nu}) = (i\bar{z})^{-1/2}\tilde{\psi}^\mu(\bar{\nu})\,, \end{aligned} \tag{A.22}$$

leads to the following Fourier expansions in torus coordinates

$$\psi^\mu(\nu) = (-i)^{1/2} \sum_{r\in\mathbb{Z}+\gamma} \psi_r^\mu e^{-2\pi i r\nu}\,, \qquad \tilde{\psi}^\mu(\bar{\nu}) = i^{1/2} \sum_{r\in\mathbb{Z}+\tilde{\gamma}} \tilde{\psi}_r^\mu e^{2\pi i r\bar{\nu}}\,. \tag{A.23}$$

Using (A.21) and (A.22), it is also easy to see that the OPEs in torus coordinates are given by

$$\psi^\mu(\nu_1)\psi^\nu(\nu_2) \sim \frac{\eta^{\mu\nu}}{\nu_1-\nu_2}\,, \qquad \tilde{\psi}^\mu(\bar{\nu}_1)\tilde{\psi}^\nu(\bar{\nu}_2) \sim \frac{\eta^{\mu\nu}}{\bar{\nu}_1-\bar{\nu}_2}\,. \tag{A.24}$$

Using

$$\vartheta_1(\nu,\tau) \xrightarrow{\nu\to 0} 2\pi\eta(\tau)^3\nu \tag{A.25}$$

makes it clear that the correlators (A.10) and (A.12) have short distance behavior consistent with the OPEs (A.24). The differing short distance behavior of the correlator (A.9) in [10] can now be understood by the redefinitions

$$\psi^{(\mathrm{ABFPT})}(\nu) = (-2i)^{-1/2}\psi^{(\mathrm{here})}(\nu)\,, \qquad \tilde{\psi}^{(\mathrm{ABFPT})}(\bar{\nu}) = (2i)^{-1/2}\tilde{\psi}^{(\mathrm{here})}(\bar{\nu})\,. \tag{A.26}$$

We proceed to construct the fermionic correlators (A.14) by images under the involutions (A.8). Consider the two Clifford algebras

$$\{\gamma_a,\gamma_b\} = \pm 2\eta_{ab}\mathbf{1}\,, \tag{A.27}$$

where η_{ab} is the world-sheet metric. For non-orientable surfaces, we must a priori consider both signs in (A.27). With fixed euclidean signature η_{ab}, that we take to be $(++)$, the two algebras generate two different groups $\mathrm{Pin}^+(2)$ and $\mathrm{Pin}^-(2)$ (see [48] for an introduction to these groups). In both cases there is a choice of (2-dimensional, chiral) representation[29] for γ_1 and γ_2. For $\mathrm{Pin}^+(2)$ the matrices square to $+\mathbf{1}$ by (A.27), so we can pick γ_1, γ_2 to be the Pauli matrices

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{A.28}$$

whereas for $\mathrm{Pin}^-(2)$ we can take

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}. \tag{A.29}$$

The involutions (A.8) precisely correspond to the parity transformation on the respective surfaces (i.e. one returns to the original point and changes the orientation, see figure 3). The corresponding reflection matrix is $P^a_b = \mathrm{diag}(-1,1)$, because in $\nu = \sigma_1 + i\sigma_2$ the world-sheet space coordinate is given by σ_1. As is familiar from four dimensions, the Lorentz matrix $\Lambda^a{}_b = P^a_b$ acting on the coordinates induces an action on the (s)pinors by

$$\begin{pmatrix} \psi(\nu) \\ \tilde{\psi}(\bar{\nu}) \end{pmatrix} \xrightarrow{P} S_P \begin{pmatrix} \psi(I_\sigma(\nu)) \\ \tilde{\psi}(I_\sigma(\bar{\nu})) \end{pmatrix} \tag{A.30}$$

[29]By chiral representation we mean one for which $\gamma_1\gamma_2$ is diagonal.

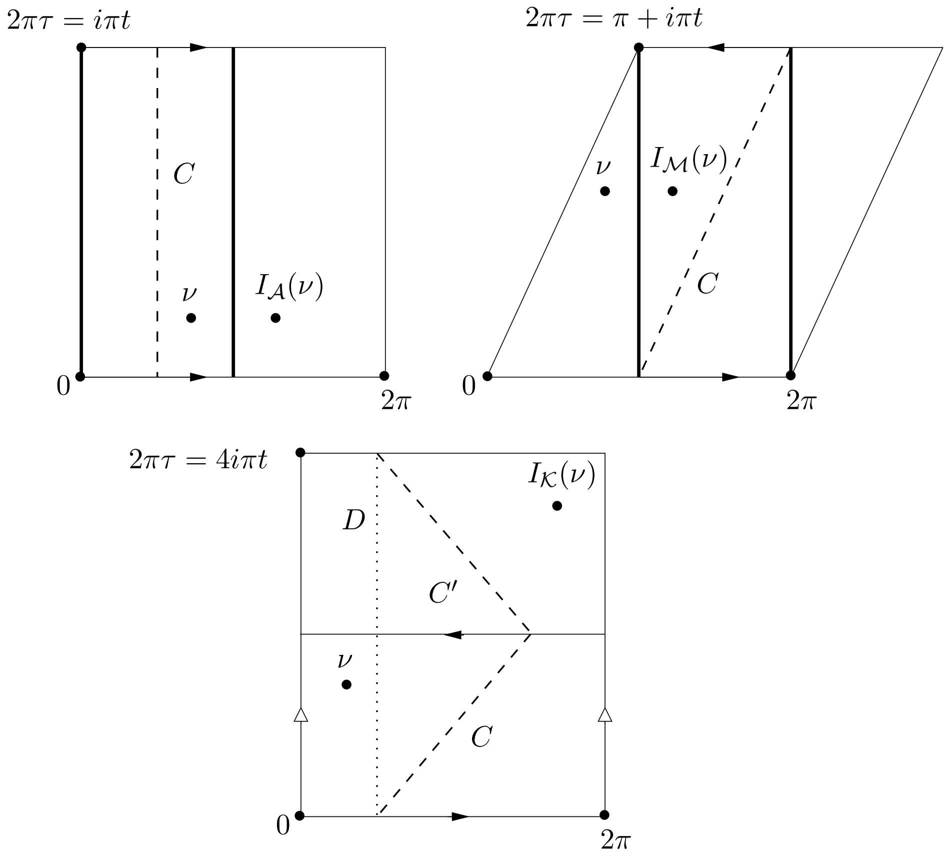

Figure 3: Annulus, Möbius strip and Klein bottle obtained from covering tori by the involutions (A.8). Thick lines are boundaries (fixed lines under the involution). Each surface has one 1-cycle C. On the Klein bottle, the equality $CC' = D$ gives rise to the constraint (A.39).

for $\sigma \in \{\mathcal{A}, \mathcal{M}, \mathcal{K}\}$, where the matrix S_P is given by

$$S_P^{-1} \gamma^a S_P = P^a_b \gamma^b \, . \tag{A.31}$$

Using the algebra (A.27), equation (A.31) yields $S_P = \pm\gamma_2$ and we take[30] $S_P = \gamma_2$. We will now argue that for the annulus and Möbius strip, γ_2 must be the second matrix in (A.28); if we were to pick the first matrix, the propagator of Majorana fermions would vanish.[31] Define the Majorana pinor

$$\Psi(\nu, \bar{\nu}) = \begin{pmatrix} \psi(\nu) \\ \tilde{\psi}(\bar{\nu}) \end{pmatrix} \tag{A.32}$$

as in [51], i.e. by demanding

$$\overline{\Psi}_{\mathrm{D}} = \Psi^\dagger \gamma_2 \stackrel{!}{=} \Psi^T C = \overline{\Psi}_{\mathrm{M}} \, , \tag{A.33}$$

[30]The overall sign is a matter of convention; changing it merely maps the pin structures into each other. An overall factor of i changes the algebra.

[31]For the annulus and Möbius strip, we pick Pin^+ in our conventions, whereas for the Klein bottle we need to allow both Pin^+ and Pin^-, as expressed in eq. (A.39) below. This choice is in accord with [10]. It would be interesting to work out precisely how the topological obstructions in [49, 50] manifest themselves here.

where $\overline{\Psi}_{\rm D}$ is the Dirac adjoint, $\overline{\Psi}_{\rm M}$ is the Majorana conjugate and C is the charge conjugation matrix, whose explicit form we do not need in the following. Then the Dirac propagator on the torus is given by [10]

$$\begin{aligned}\langle\Psi(\nu_1,\bar\nu_1)\overline{\Psi}_{\rm D}(\nu_2,\bar\nu_2)\rangle_{\mathcal{T}} &= \langle\Psi(\nu_1,\bar\nu_1)\Psi^T(\nu_2,\bar\nu_2)\rangle_{\mathcal{T}}C \\ &= \begin{pmatrix}\langle\psi(\nu_1)\psi(\nu_2)\rangle_{\mathcal{T}}^{\alpha,\beta} & 0 \\ 0 & \langle\tilde\psi(\bar\nu_1)\tilde\psi(\bar\nu_2)\rangle_{\mathcal{T}}^{\bar\alpha,\bar\beta}\end{pmatrix} C\,. \end{aligned} \tag{A.34}$$

The propagators on the other surfaces are determined by symmetrizing (A.34) with respect to the involutions (A.8), producing

$$\begin{aligned}\langle\Psi(\nu_1)\overline{\Psi}_{\rm D}(\nu_2)\rangle_\sigma &= \frac{1}{2}\Big(\langle\Psi(\nu_1)\Psi^T(\nu_2)\rangle_{\mathcal{T}} + \gamma_2\langle\Psi(I_\sigma(\nu_1))\Psi^T(\nu_2)\rangle_{\mathcal{T}} + \\ &\quad + \langle\Psi(\nu_1)\Psi^T(I_\sigma(\nu_2))\rangle_{\mathcal{T}}\gamma_2^T + \gamma_2\langle\Psi(I_\sigma(\nu_1))\Psi^T(I_\sigma(\nu_2))\rangle_{\mathcal{T}}\gamma_2^T\Big)C\,,\end{aligned} \tag{A.35}$$

with $\sigma\in\{\mathcal{A},\mathcal{M},\mathcal{K}\}$. To go further, we have to distinguish between annulus and Möbius strip on the one hand and Klein bottle on the other. For $\sigma=\mathcal{A},\mathcal{M}$, using (A.34) and (A.28), the propagator (A.35) becomes

$$\frac{1}{2}\begin{bmatrix} P_F(s,\nu_1,\nu_2)-\overline{P_F(\bar s,I_\sigma(\nu_1),I_\sigma(\nu_2))} & i(P_F(s,\nu_1,I_\sigma(\nu_2))-\overline{P_F(\bar s,I_\sigma(\nu_1),\nu_2)}) \\ i(P_F(s,I_\sigma(\nu_1),\nu_2)-\overline{P_F(\bar s,\nu_1,I_\sigma(\nu_2))}) & \overline{P_F(\bar s,\nu_1,\nu_2)}-P_F(s,I_\sigma(\nu_1),I_\sigma(\nu_2))\end{bmatrix}C\,. \tag{A.36}$$

For the annulus, the spin structures for left- and right-movers are the same on the covering torus ($s=\bar s$), and the complex structure modulus of the covering torus is purely imaginary. Then, from the definition of the theta functions in [32] we see that $\overline{P_F(s,\nu_1,\nu_2)}=P_F(s,\bar\nu_1,\bar\nu_2)=-P_F(s,-\bar\nu_1,-\bar\nu_2)$. On the other hand, for the Möbius strip, $(s,\bar s)=(2,2),(3,4),(4,3)$ for the even spin structures, cf. [10]. The Möbius complex structure has real part $1/2$, so that

$$\begin{aligned}\overline{\left(\frac{\vartheta_{3/4}(\frac{\nu_1-\nu_2}{2\pi},\tau)}{\vartheta_{3/4}(0,\tau)}\right)_{\mathcal{M}}} &= \left(\frac{\vartheta_{4/3}(\frac{\bar\nu_1-\bar\nu_2}{2\pi},\tau)}{\vartheta_{4/3}(0,\tau)}\right)_{\mathcal{M}}\,, \\ \overline{\left(\frac{\vartheta_{2}(\frac{\nu_1-\nu_2}{2\pi},\tau)}{\vartheta_{2}(0,\tau)}\right)_{\mathcal{M}}} &= \left(\frac{\vartheta_{2}(\frac{\bar\nu_1-\bar\nu_2}{2\pi},\tau)}{\vartheta_{2}(0,\tau)}\right)_{\mathcal{M}}\,, \\ \overline{\left(\frac{\vartheta_{1}(\frac{\nu_1-\nu_2}{2\pi},\tau)}{\vartheta_{1}'(0,\tau)}\right)_{\mathcal{M}}} &= \left(\frac{\vartheta_{1}(\frac{\bar\nu_1-\bar\nu_2}{2\pi},\tau)}{\vartheta_{1}'(0,\tau)}\right)_{\mathcal{M}}\,.\end{aligned} \tag{A.37}$$

We arrive at

$$\langle\Psi(\nu_1)\overline{\Psi}_{\rm D}(\nu_2)\rangle_\sigma = \begin{pmatrix} P_F(s,\nu_1,\nu_2) & iP_F(s,\nu_1,I_\sigma(\nu_2)) \\ iP_F(s,I_\sigma(\nu_1),\nu_2) & \overline{P_F(\bar s,\nu_1,\nu_2)}\end{pmatrix}C\,. \tag{A.38}$$

(Recall that up to now, this only holds for $\sigma=\mathcal{A},\mathcal{M}$; however, we will see in a moment that the same holds also for $\sigma=\mathcal{K}$.) Note that if we had chosen γ_2 as the first matrix in (A.28), the propagator had come out to be identically zero. That is why we have to choose γ_2 as the second matrix in (A.28) for our conventions of fermions.[32]

We now proceed to show that the same correlators also hold for the Klein bottle $\mathcal{K}$. To see this, first note that there is a complication that only appears for $\mathcal{K}$ and not for $\mathcal{A}, \mathcal{M}$. Referring to figure 3, we see that going twice around the Klein-bottle cycle C (which is equivalent to first traversing C, then C') has the same effect as following the path D on the covering torus. This puts a constraint on the action of the square of parity S_P^2 on the pinors, depending on their spin structure on the covering torus. More precisely, in (A.30) we need to introduce different S_P for different spin structures, satisfying

$$\left(S_P\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\right)^2 = (-1)^{2\beta+1}\mathbf{1} \qquad \text{(Klein bottle only)}\,. \tag{A.39}$$

Thus, for $s=4$ and $s=1$ (again we refer to table 3), the constraint (A.39) leads to $\mathrm{Pin}^+(2)$ as in the case of the annulus and Möbius strip. For $s=2$ and $s=3$, the constraint (A.39) forces us to choose $\mathrm{Pin}^-(2)$. Retracing the above steps for $s=2$ and $s=3$ leads to the Dirac propagator

$$\begin{aligned}&\langle\Psi(\nu_1,\bar\nu_1)\overline{\Psi}_{\mathrm{D}}(\nu_2,\bar\nu_2)\rangle_{\mathcal{K}} = \qquad\qquad (\mathrm{A.40})\\ &= \frac{1}{2}\begin{bmatrix} P_F(s,\nu_1,\nu_2)-\overline{P_F(\bar s,I_{\mathcal{K}}(\nu_1),I_{\mathcal{K}}(\nu_2))} & i(P_F(s,\nu_1,I_{\mathcal{K}}(\nu_2))+\overline{P_F(\bar s,I_{\mathcal{K}}(\nu_1),\nu_2)}) \\ i(P_F(s,I_{\mathcal{K}}(\nu_1),\nu_2)+\overline{P_F(\bar s,\nu_1,I_{\mathcal{K}}(\nu_2))}) & \overline{P_F(\bar s,\nu_1,\nu_2)}-P_F(s,I_{\mathcal{K}}(\nu_1),I_{\mathcal{K}}(\nu_2))\end{bmatrix}C\,.\end{aligned}$$

For the Klein bottle, like for the annulus, the spin structures for left and right movers are the same on the covering torus ($s=\bar s$) and the complex structure of the covering torus is purely imaginary, so again $\overline{P_F(s,\nu_1,\nu_2)} = -P_F(s,-\bar\nu_1,-\bar\nu_2)$. For the off-diagonal elements, one can show that the periodicity (A.16) implies

$$\begin{aligned}\overline{P_F(\bar s,I_{\mathcal{K}}(\nu_1),\nu_2)} &= P_F(s,\nu_1,I_{\mathcal{K}}(\nu_2))\,, \qquad s=2,3\,,\\ \overline{P_F(\bar s,I_{\mathcal{K}}(\nu_1),\nu_2)} &= -P_F(s,\nu_1,I_{\mathcal{K}}(\nu_2))\,, \qquad s=4\,.\end{aligned} \tag{A.41}$$

Substituting this into (A.40) one ends up with the same results as in (A.38) above also for $s=2,3$, and hence for all even spin structures $s=2,3,4$. This concludes the derivation of the correlators summarized in (A.14).

It may be useful to note that had we blindly tried to use the correlators of [10] without adjusting to our conventions, the "torus trick" in equation (2.62) would not work. Also see [52 − 54] for some related discussion.

B. Some properties of $E_2(A,U)$

In this appendix we discuss two issues related to the function $E_2(A,U)$ which we defined in (2.65). We show how to rewrite it in terms of polylogarithms, which is useful for expressing the one-loop correction to the Kähler potential in terms of a prepotential. Furthermore, we display its transformation property under $\mathrm{SL}(2,\mathbb{Z})$ transformations of its arguments, i.e. under T-duality.

[32] For the conventions of [10], one has for instance for the annulus $\overline{P_F(s,\nu_1,\nu_2)} = -P_F(s,\bar\nu_1,\bar\nu_2)$, as there is an additional i in the definition of P_F, cf. their (A.9). Thus, for them the first matrix in (A.28) is the right choice for γ_2, which is what they report in (A.10).

B.1 Writing $E_2(A,U)$ in terms of polylogarithms

The goal is to rewrite $E_2(A,U)$ such that $U_2E_2(A,U)$ can be expressed in a form like the argument of the logarithm in (2.14) via some holomorphic prepotential $\mathcal{F}(A,U)$. This can be derived in the same way as (1.4) of [55]. We start with $E_2(A,U)$ defined in (2.65), split off the term with $m=0$, in the rest use

$$y^{-s} = \frac{1}{\Gamma(s)}\int_0^\infty dx\, x^{s-1}e^{-yx}\,. \tag{B.1}$$

Next we perform a Poisson resummation over n and then split off the term with $n=0$. One also has to use

$$\int_0^\infty dx\, x^{\nu-1}e^{-\beta/x-\gamma x} = 2\left(\frac{\beta}{\gamma}\right)^{\nu/2} K_\nu(2\sqrt{\beta\gamma})\,, \qquad \mathrm{Re}(\beta)>0\,, \quad \mathrm{Re}(\gamma)>0\,. \tag{B.2}$$

Doing so one derives

$$\begin{aligned}
E_2(A,U) \,=\, & 2U_2^2\pi^4\left(\frac{1}{90}-\frac{1}{3}\frac{A_2^2}{U_2^2}+\frac{2}{3}\frac{A_2^3}{U_2^3}-\frac{1}{3}\frac{A_2^4}{U_2^4}\right)+\frac{\pi}{2U_2}\Big[Li_3(e^{2\pi iA})+Li_3(e^{-2\pi i\bar{A}})\Big]+\\
&+\pi^2\frac{A_2}{U_2}\Big[Li_2(e^{2\pi iA})+Li_2(e^{-2\pi i\bar{A}})\Big]+2\pi^2\sqrt{U_2}\sum_{m\neq 0,n\neq 0}\left|\frac{m-\frac{A_2}{U_2}}{n}\right|^{3/2}\times\\
&\times K_{3/2}\left(2\pi\left|nm-n\frac{A_2}{U_2}\right|U_2\right)e^{2\pi in(mU_1-A_1)}\,,
\end{aligned} \tag{B.3}$$

using

$$K_{3/2}(z) = \left(\frac{\pi}{2z}\right)^{1/2}e^{-z}(1+z^{-1}) \tag{B.4}$$

and

$$Li_n(z) = \sum_{k=1}^\infty \frac{z^k}{k^n}\,. \tag{B.5}$$

Using (B.4) again, this can be rewritten as (assuming $a_1 = A_2/U_2 < 1$)

$$\begin{aligned}
E_2(A,U) \,=\, & 2U_2^2\pi^4\left(\frac{1}{90}-\frac{1}{3}\frac{A_2^2}{U_2^2}+\frac{2}{3}\frac{A_2^3}{U_2^3}-\frac{1}{3}\frac{A_2^4}{U_2^4}\right)+\\
&+\frac{\pi}{2U_2}\Big[Li_3(e^{2\pi iA})+Li_3(e^{-2\pi i\bar{A}})\Big]+\pi^2\frac{A_2}{U_2}\Big[Li_2(e^{2\pi iA})+Li_2(e^{-2\pi i\bar{A}})\Big]+\\
&+\pi^2\sum_{m>0}\left(m-\frac{A_2}{U_2}\right)\Big[Li_2(e^{2\pi i(mU-A)})+Li_2(e^{-2\pi i(m\bar{U}-\bar{A})})\Big]+\\
&+\pi^2\sum_{m>0}\left(m+\frac{A_2}{U_2}\right)\Big[Li_2(e^{2\pi i(mU+A)})+Li_2(e^{-2\pi i(m\bar{U}+\bar{A})})\Big]+\\
&+\frac{\pi}{2U_2}\sum_{m>0}\Big[Li_3(e^{2\pi i(mU-A)})+Li_3(e^{-2\pi i(m\bar{U}-\bar{A})})+\\
&\qquad+Li_3(e^{2\pi i(mU+A)})+Li_3(e^{-2\pi i(m\bar{U}+\bar{A})})\Big]\,.
\end{aligned} \tag{B.6}$$

This formula is essential to derive (2.81).

B.2 The $\mathrm{SL}(2,\mathbb{Z})$ transformation of $E_2(A,U)$

We now show how $E_2(A,U)$ transforms under $\mathrm{SL}(2,\mathbb{Z})$. Define

$$Z(\vec{a},U) = \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i \vec{n}\vec{a}}}{|n+mU|^4}\,, \tag{B.7}$$

where $\vec{a}$ is a real vector. More precisely, it determines a position $a_1 e^1_M + a_2 e^2_M$ on a torus, whose lattice is defined by the vielbein e^m_M. Now perform an $\mathrm{SL}(2,\mathbb{Z})$ transformation

$$U \;\rightarrow\; \frac{AU+B}{CU+D}\,, \qquad a_m \;\rightarrow a^m \;\;, \tag{B.8}$$

where

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) \tag{B.9}$$

and a^m defines a position $a^1 e_1^M + a^2 e_2^M$ on the transformed lattice that we will again, by slight abuse of notation, denote by $\vec{a}$. Now (B.7) transforms under (B.8) according to

$$\begin{aligned} Z(\vec{a},U) \rightarrow & \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i \vec{n}\vec{a}}}{|n+\frac{AU+B}{CU+D}m|^4} \\ = & \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i \vec{n}\vec{a}}}{|Dn+Bm+(Am+Cn)U|^4}|CU+D|^4\,. \end{aligned} \tag{B.10}$$

Introducing new variables

$$\tilde{n} = Dn + Bm\,, \qquad \tilde{m} = Am + Cn \tag{B.11}$$

or

$$n = A\tilde{n} - B\tilde{m}\,, \qquad m = D\tilde{m} - C\tilde{n}\,, \tag{B.12}$$

and using

$$\vec{n}\vec{a} = \tilde{n}(Aa^1 - Ca^2) + \tilde{m}(-a^1 B + a^2 D)\,, \tag{B.13}$$

we arrive at

$$Z\Big(\vec{a}, \frac{AU+B}{CU+D}\Big) = |CU+D|^4 Z(\tilde{\vec{a}},U)\,, \tag{B.14}$$

with

$$\tilde{\vec{a}} = \begin{pmatrix} \tilde{a}^1 \\ \tilde{a}^2 \end{pmatrix} = \begin{pmatrix} A & -C \\ -B & D \end{pmatrix}\begin{pmatrix} a^1 \\ a^2 \end{pmatrix}. \tag{B.15}$$

On the other hand

$$U_2^2 \;\rightarrow\; U_2^2 |CU+D|^{-4}\,. \tag{B.16}$$

Taking (B.14) and (B.16) together, we arrive at

$$E_2(A,U) \;\rightarrow\; E_2(\tilde{A},U)\,, \tag{B.17}$$

where

$$\tilde{A} = U\tilde{a}^1 - \tilde{a}^2\,. \tag{B.18}$$

Equation (B.17) is the desired result. As a special case for $A=0$, we recover $\mathrm{SL}(2,\mathbb{Z})$ invariance of the nonholomorphic Eisenstein series $E_2(0,U)$.

C. The other one-loop 2-point functions

In this appendix we compute the remaining 2-point functions for the Kähler variables $\{S', U, A_i\}$ of the $\mathcal{N}=2$ orientifold discussed in section 2, where only the 2-point function $\langle V_{S_2'} V_{S_2'}\rangle$ was considered. Doing so, we confirm the formula (2.77) for the Kähler potential that was the main result of section 2.

Let us start by giving the metric components derived from the Kähler potential (2.77) that we would like to reproduce by the 2-point functions. Up to one-loop order they are given by

$$
\begin{aligned}
K_{U\bar{U}} &= K^{(0)}_{U\bar{U}} + \frac{c}{(S-\bar{S})(S_0'-\bar{S}_0')}\partial_U\partial_{\bar{U}}\mathcal{E}_2(A_k,U) + \mathcal{O}(e^{3\Phi})\,,\\
K_{A_i\bar{A}_j} &= K^{(0)}_{A_i\bar{A}_j} + \frac{c}{(S-\bar{S})(S_0'-\bar{S}_0')}\partial_{A_i}\partial_{\bar{A}_j}\mathcal{E}_2(A_k,U) + \mathcal{O}(e^{3\Phi})\,,\\
K_{U\bar{S}'} &= K^{(0)}_{U\bar{S}'} + \frac{c}{(S-\bar{S})(S_0'-\bar{S}_0')^2}\partial_U\mathcal{E}_2(A_k,U) + \mathcal{O}(e^{4\Phi})\,,\\
K_{U\bar{A}_i} &= K^{(0)}_{U\bar{A}_i} + \frac{c}{(S-\bar{S})(S_0'-\bar{S}_0')}\partial_U\partial_{\bar{A}_i}\mathcal{E}_2(A_k,U) + \mathcal{O}(e^{3\Phi})\,,\\
K_{A_i\bar{S}'} &= K^{(0)}_{A_i\bar{S}'} + \frac{c}{(S-\bar{S})(S_0'-\bar{S}_0')^2}\partial_{A_i}\mathcal{E}_2(A_k,U) + \mathcal{O}(e^{4\Phi})\,,
\end{aligned}
\tag{C.1}
$$

where the terms with a superscript (0) were already given in (2.16). The 2-point functions that we want to compute are

$$
\begin{aligned}
\langle V_U V_{\bar{U}}\rangle &= -\sum_\sigma \frac{4g_c^2\alpha'^{-4}}{(U-\bar{U})^2}\langle V^{(0,0)}_{ZZ}V^{(0,0)}_{\bar{Z}\bar{Z}}\rangle_\sigma +\\
&\quad+\sum_\sigma\Bigg[\sum_B \frac{8\pi g_c^2\alpha'^{-4}(A_{s[B]}-\bar{A}_{s[B]})}{(U-\bar{U})^{5/2}(T-\bar{T})^{1/2}}\left(\langle V^{(0,0)}_{ZZ}V^{(0)B}_{\bar{Z}}\rangle_\sigma + \langle V^{(0)B}_{Z}V^{(0,0)}_{\bar{Z}\bar{Z}}\rangle_\sigma\right) -\\
&\qquad -\sum_{B,C}\frac{16\pi^2 g_c^2\alpha'^{-4}(A_{s[B]}-\bar{A}_{s[B]})(A_{s[C]}-\bar{A}_{s[C]})}{(U-\bar{U})^3(T-\bar{T})}\langle V^{(0)B}_Z V^{(0)C}_{\bar{Z}}\rangle_\sigma\Bigg] + \mathcal{O}(e^\Phi)\,,\\
\langle V_{A_i}V_{\bar{A}_j}\rangle &= -\sum_\sigma\sum_{B,C}\frac{16 g_o^2\alpha'^{-4}\delta_{is[B]}\delta_{js[C]}}{(U-\bar{U})(T-\bar{T})}\langle V^{(0)B}_Z V^{(0)C}_{\bar{Z}}\rangle_\sigma + \mathcal{O}(e^\Phi)\,,\\
\langle V_U V_{S_2'}\rangle &= -\sum_\sigma\frac{i2g_c^2\alpha'^{-4}}{(U-\bar{U})(S_0'-\bar{S}_0')}\langle V^{(0,0)}_{ZZ}V^{(0,0)}_{Z\bar{Z}}\rangle_\sigma +\\
&\quad+\sum_\sigma\sum_B\frac{i4\pi g_c^2\alpha'^{-4}(A_{s[B]}-\bar{A}_{s[B]})}{(U-\bar{U})^{3/2}(T-\bar{T})^{1/2}(S_0'-\bar{S}_0')}\langle V^{(0)B}_Z V^{(0,0)}_{Z\bar{Z}}\rangle_\sigma + \mathcal{O}(e^{2\Phi})\,,\\
\langle V_U V_{\bar{A}_i}\rangle &= \sum_\sigma\Bigg[-\sum_B\frac{8g_c g_o\alpha'^{-4}\delta_{is[B]}}{(U-\bar{U})^{3/2}(T-\bar{T})^{1/2}}\langle V^{(0,0)}_{ZZ}V^{(0)B}_{\bar{Z}}\rangle_\sigma +\\
&\qquad+\sum_{B,C}\frac{16\pi g_c g_o\alpha'^{-4}(A_{s[B]}-\bar{A}_{s[B]})\delta_{is[C]}}{(U-\bar{U})^2(T-\bar{T})}\langle V^{(0)B}_Z V^{(0)C}_{\bar{Z}}\rangle_\sigma\Bigg] + \mathcal{O}(e^\Phi)\,,\\
\langle V_{A_i}V_{S_2'}\rangle &= -\sum_\sigma\sum_B\frac{i4g_c g_o\alpha'^{-4}\delta_{is[B]}}{(U-\bar{U})^{1/2}(T-\bar{T})^{1/2}(S_0'-\bar{S}_0')}\langle V^{(0)B}_Z V^{(0,0)}_{Z\bar{Z}}\rangle_\sigma + \mathcal{O}(e^{2\Phi})\,.
\end{aligned}
\tag{C.2}
$$

The summation over σ effectively only runs over $\{(ij), (ia), (ai)\}$ and $\{(i)\}$ for the correlators involving the open string vertex operators $V^{(0)B}$, because we are only considering Wilson line moduli on the 9-branes. We have given only the leading contribution in an expansion in the dilaton. This is the only one that we surely have to be able to reproduce with the Kähler potential that we suggested in 2.6. Higher order terms (arising from the higher order corrections in the vertex operators (2.34)) might in general also get contributions from higher genus world-sheets in string perturbation theory. We do not attempt to calculate these higher order terms here. All amplitudes are expressed by the following basic correlators

$$\begin{aligned}
\langle V^{(0,0)}_{ZZ} V^{(0,0)}_{\bar{Z}\bar{Z}} \rangle_\sigma &= -V_4 \frac{(p_1 \cdot p_2)\sqrt{G}}{16(4\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^4} \int_{\mathcal{F}_\sigma} d^2\nu_1 d^2\nu_2 \times \\
&\quad \times \sum_{k=0,1} \sum_{\vec{n}=(n,m)^T} \mathrm{tr} \Bigg[e^{-\pi \vec{n}^T G \vec{n} t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma \cdot \vec{n}} \sum_{\substack{\alpha\beta \\ \text{even}}} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)} \gamma_{\sigma,k} \mathcal{Z}^{\mathrm{int}}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \times \\
&\quad \times \Big[\langle \bar{\partial} Z(\bar{\nu}_1) \bar{\partial} \bar{Z}(\bar{\nu}_2) \rangle_\sigma \langle \Psi(\nu_1) \bar{\Psi}(\nu_2) \rangle^{\alpha,\beta}_\sigma \langle \psi(\nu_1) \psi(\nu_2) \rangle^{\alpha,\beta}_\sigma + \\
&\quad + \langle \bar{\partial} Z(\bar{\nu}_1) \partial \bar{Z}(\nu_2) \rangle_\sigma \langle \Psi(\nu_1) \bar{\tilde{\bar{\Psi}}}(\bar{\nu}_2) \rangle^{\alpha,\beta}_\sigma \times \\
&\quad \times \langle \psi(\nu_1) \tilde{\psi}(\bar{\nu}_2) \rangle^{\alpha,\beta}_\sigma + \text{c.c.} \Big] \Bigg] + \mathcal{O}(\delta^2)\,, \qquad \text{(C.3)}
\end{aligned}$$

$$\begin{aligned}
\langle V^{(0)B}_{Z} V^{(0)C}_{\bar{Z}} \rangle_\sigma &= -V_4 \frac{(p_1 \cdot p_2)\alpha'\sqrt{G}}{8(4\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^4} \int_{(\partial\Sigma)_B} d\tilde{\nu}_1 \int_{(\partial\Sigma)_C} d\tilde{\nu}_2 \times \\
&\quad \times \sum_{k=0,1} \sum_{\vec{n}=(n,m)^T} \mathrm{tr} \Bigg[\lambda_{s[B]} \lambda^\dagger_{s[C]} e^{-\pi \vec{n}^T G \vec{n} t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma \cdot \vec{n}} \times \\
&\quad \times \sum_{\substack{\alpha\beta \\ \text{even}}} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)} \gamma_{\sigma,k} \mathcal{Z}^{\mathrm{int}}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \times \\
&\quad \times \langle \Psi(\nu_1) \bar{\Psi}(\nu_2) \rangle^{\alpha,\beta}_\sigma \langle \psi(\nu_1) \psi(\nu_2) \rangle^{\alpha,\beta}_\sigma \Bigg] + \mathcal{O}(\delta^2)\,, \qquad \text{(C.4)}
\end{aligned}$$

$$\begin{aligned}
\langle V^{(0,0)}_{ZZ} V^{(0,0)}_{\bar{Z}\bar{Z}} \rangle_\sigma &= -V_4 \frac{(p_1 \cdot p_2)\sqrt{G}}{16(4\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^4} \int_{\mathcal{F}_\sigma} d^2\nu_1 d^2\nu_2 \times \\
&\quad \times \sum_{k=0,1} \sum_{\vec{n}=(n,m)^T} \mathrm{tr} \Bigg[e^{-\pi \vec{n}^T G \vec{n} t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma \cdot \vec{n}} \sum_{\substack{\alpha\beta \\ \text{even}}} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)} \gamma_{\sigma,k} \mathcal{Z}^{\mathrm{int}}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \times \\
&\quad \times \Big[\langle \bar{\partial} Z(\bar{\nu}_1) \bar{\partial} Z(\bar{\nu}_2) \rangle_\sigma \langle \bar{\Psi}(\nu_1) \Psi(\nu_2) \rangle^{\alpha,\beta}_\sigma \langle \psi(\nu_1) \psi(\nu_2) \rangle^{\alpha,\beta}_\sigma + \\
&\quad + \langle \partial Z(\nu_1) \bar{\partial} Z(\bar{\nu}_2) \rangle_\sigma \langle \bar{\tilde{\bar{\Psi}}}(\bar{\nu}_1) \Psi(\nu_2) \rangle^{\alpha,\beta}_\sigma \times \\
&\quad \times \langle \tilde{\psi}(\bar{\nu}_1) \psi(\nu_2) \rangle^{\alpha,\beta}_\sigma + \text{c.c.} \Big] \Bigg] + \mathcal{O}(\delta^2)\,, \qquad \text{(C.5)}
\end{aligned}$$

$$
\begin{aligned}
\langle V^{(0,0)}_{Z\bar{Z}} V^{(0)B}_{\bar{Z}} \rangle_\sigma \;=\; & -iV_4 \frac{(p_1\cdot p_2)\sqrt{2\alpha'}\sqrt{G}}{16(4\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^4} \int_{\mathcal{F}_\sigma} d^2\nu_1 \int_{(\partial\Sigma)_B} d\tilde{\nu}_2 \times \\
& \times \sum_{k=0,1} \sum_{\vec{n}=(n,m)^T} \mathrm{tr}\left[\lambda^\dagger_{s[B]} e^{-\pi \vec{n}^T G \vec{n} t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma\cdot\vec{n}} \sum_{\substack{\alpha\beta \\ \text{even}}} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)} \gamma_{\sigma,k} \mathcal{Z}^{\text{int}}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \times \right. \\
& \qquad\qquad \times \Big[\langle i\bar{\partial} Z(\bar{\nu}_1)\rangle_\sigma \langle \Psi(\nu_1)\bar{\Psi}(\nu_2)\rangle^{\alpha,\beta}_\sigma \langle \psi(\nu_1)\psi(\nu_2)\rangle^{\alpha,\beta}_\sigma + \\
& \qquad\qquad + \langle i\partial Z(\nu_1)\rangle_\sigma \langle \tilde{\Psi}(\bar{\nu}_1)\bar{\Psi}(\nu_2)\rangle^{\alpha,\beta}_\sigma \times \\
& \qquad\qquad \left. \times \langle \tilde{\psi}(\bar{\nu}_1)\psi(\nu_2)\rangle^{\alpha,\beta}_\sigma \Big] \right] + \mathcal{O}(\delta^2)\,, \qquad \text{(C.6)}
\end{aligned}
$$

$$
\begin{aligned}
\langle V^{(0)B}_{Z} V^{(0,0)}_{Z\bar{Z}} \rangle_\sigma \;=\; & -iV_4 \frac{(p_1\cdot p_2)\sqrt{2\alpha'}\sqrt{G}}{16(4\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^4} \int_{(\partial\Sigma)_B} d\tilde{\nu}_1 \int_{\mathcal{F}_\sigma} d^2\nu_2 \times \\
& \times \sum_{k=0,1} \sum_{\vec{n}=(n,m)^T} \mathrm{tr}\left[\lambda_{s[B]} e^{-\pi \vec{n}^T G \vec{n} t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma\cdot\vec{n}} \sum_{\substack{\alpha\beta \\ \text{even}}} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)} \gamma_{\sigma,k} \mathcal{Z}^{\text{int}}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \times \right. \\
& \qquad\qquad \times \Big[\langle i\bar{\partial} Z(\bar{\nu}_2)\rangle_\sigma \langle \Psi(\nu_1)\bar{\Psi}(\nu_2)\rangle^{\alpha,\beta}_\sigma \langle \psi(\nu_1)\psi(\nu_2)\rangle^{\alpha,\beta}_\sigma + \\
& \qquad\qquad + \langle i\partial Z(\nu_2)\rangle_\sigma \langle \Psi(\nu_1)\tilde{\bar{\Psi}}(\bar{\nu}_2)\rangle^{\alpha,\beta}_\sigma \times \\
& \qquad\qquad \left. \times \langle \psi(\nu_1)\tilde{\psi}(\bar{\nu}_2)\rangle^{\alpha,\beta}_\sigma \Big] \right] + \mathcal{O}(\delta^2)\,. \qquad \text{(C.7)}
\end{aligned}
$$

The last three correlation functions obviously only get contributions from the Z-zero modes (2.49), (note that the comment after (2.48), concerning the sum over zero modes, also applies for formulas (C.3)–(C.7)). Another comment is in order here, concerning the minus sign in (C.4) and the factors of i in (C.6) and (C.7). When we change coordinates in the open string amplitudes via $\theta = e^{-i\nu}$, the boundary that ran along the real axis before now runs along the imaginary axis, i.e. $\nu = i\tilde{\nu}$. If we then change the integration variable from ν to $\tilde{\nu}$ and define the derivative $\dot{Z}$ in the vertex operator of the open string according to $\dot{Z} = \partial_{\tilde{\nu}} Z$, the open string vertex operators become

$$
\begin{aligned}
V^{(0)B}_{Z} &= \frac{1}{\sqrt{2\alpha'}} \lambda_{s[B]} \int_{(\partial\Sigma)_B} d\tilde{\nu} [i\dot{Z} + i2\alpha'(p\cdot\psi)\Psi] e^{ipX}\,, \\
V^{(0)B}_{\bar{Z}} &= \frac{1}{\sqrt{2\alpha'}} \lambda^\dagger_{s[B]} \int_{(\partial\Sigma)_B} d\tilde{\nu}\; [i\dot{\bar{Z}} + i2\alpha'(p\cdot\psi)\bar{\Psi}] e^{ipX}\,, \qquad \text{(C.8)}
\end{aligned}
$$

in particular there is a factor of i now also in front of the fermionic terms.

Following the same steps as in section 2.5, we see that the theta functions from the internal partition function and the fermionic world-sheet correlators again drop out due to (2.58). The remaining bosonic world-sheet correlators can be dealt with analogously as in the main text using (2.61) and

$$
\int_{\mathcal{F}_\sigma} d^2\nu_1 d^2\nu_2 \Big[\langle \bar{\partial} Z(\bar{\nu}_1) \bar{\partial} Z(\bar{\nu}_2)\rangle_\sigma - \langle \partial Z(\nu_1) \bar{\partial} Z(\bar{\nu}_2)\rangle_\sigma + \text{c.c.} \Big] =
$$

JHEP11(2005)030

$$= -2\pi^4 c_\sigma^2 \frac{T_2}{U_2}(n+m\bar{U})^2\alpha' = \begin{cases} -2\pi^4 \frac{T_2}{U_2}(n+m\bar{U})^2\alpha' & \text{for } \mathcal{A}, \mathcal{M} \\ -8\pi^4 \frac{T_2}{U_2}(n+m\bar{U})^2\alpha' & \text{for } \mathcal{K} \end{cases}, \tag{C.9}$$

$$\int_{\mathcal{F}_\sigma} d^2\nu_1 \int_{\partial\Sigma} d\tilde{\nu}_2 \Big[\langle i\bar{\partial}Z(\bar{\nu}_1)\rangle_\sigma - \langle i\partial Z(\nu_1)\rangle_\sigma\Big] =$$

$$= -2\pi^3 c_\sigma^3 d_\sigma \sqrt{\frac{T_2}{2U_2}}(n+m\bar{U})t\sqrt{\alpha'} = \begin{cases} -2\pi^3\sqrt{\frac{T_2}{2U_2}}(n+m\bar{U})t\sqrt{\alpha'} & \text{for } \mathcal{A} \\ -4\pi^3\sqrt{\frac{T_2}{2U_2}}(n+m\bar{U})t\sqrt{\alpha'} & \text{for } \mathcal{M} \\ 0 & \text{for } \mathcal{K} \end{cases}, \tag{C.10}$$

which only get contributions from the zero modes. In (C.10) we had to introduce another constant d_σ. For the sake of completeness let us here list all three constants c_σ, d_σ and e_σ that we use in various places throughout this paper,

$$c_\sigma = \begin{cases} 1 & \text{for } \mathcal{A} \\ 1 & \text{for } \mathcal{M} \\ 2 & \text{for } \mathcal{K} \end{cases}, \qquad d_\sigma = \begin{cases} 1 & \text{for } \mathcal{A} \\ 2 & \text{for } \mathcal{M} \\ 0 & \text{for } \mathcal{K} \end{cases}, \qquad e_\sigma = \begin{cases} 1 & \text{for } \mathcal{A} \\ 4 & \text{for } \mathcal{M} \\ 4 & \text{for } \mathcal{K} \end{cases}. \tag{C.11}$$

The integrals over the world-sheet modulus t can now be performed similarly to (2.64) where we have to regulate the integrals again with a UV cutoff Λ and, where necessary, with an IR cutoff χ (UV and IR referring to the open string channel),

$$\int_0^\infty \frac{dt}{t^4} \sum_{\vec{n}=(n,m)^T} e^{-\pi\vec{n}^T G\vec{n}t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma\cdot\vec{n}} \Big(-2\pi^4 c_\sigma^2 \alpha' \frac{T_2}{U_2}(n+m\bar{U})^2\Big) =$$

$$= -4\pi c_\sigma^2 \alpha' \frac{U_2^2}{T_2^2} {\sum_{\vec{n}=(n,m)^T}}' \frac{e^{2\pi i \vec{\mathbf{A}}_\sigma\cdot\vec{n}}}{(n+mU)^3(n+m\bar{U})}, \tag{C.12}$$

$$\int_0^\infty \frac{dt}{t^3} \sum_{\vec{n}=(n,m)^T} e^{-\pi\vec{n}^T G\vec{n}t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma\cdot\vec{n}} \Big(-2\pi^3 c_\sigma^3 d_\sigma \sqrt{\alpha'}\sqrt{\frac{T_2}{2U_2}}(n+m\bar{U})\Big) =$$

$$= -\sqrt{2}\pi c_\sigma^3 d_\sigma \sqrt{\alpha'} \frac{U_2^{3/2}}{T_2^{3/2}} {\sum_{\vec{n}=(n,m)^T}}' \frac{e^{2\pi i \vec{\mathbf{A}}_\sigma\cdot\vec{n}}}{|n+mU|^4}(n+m\bar{U}), \tag{C.13}$$

$$\int_{1/(e_\sigma\Lambda^2)}^\infty \frac{dt}{t^2} \sum_{\vec{n}=(n,m)^T} e^{-\pi\vec{n}^T G\vec{n}t^{-1}} e^{2\pi i \vec{\mathbf{A}}_\sigma\cdot\vec{n}} \Big(\pi^2 d_\sigma^2 e^{-2\pi\chi t}\Big) =$$

$$= \pi^2 d_\sigma^2 e_\sigma \Lambda^2 + \pi d_\sigma^2 T_2^{-1} \tilde{E}_1(\mathbf{A}_\sigma, U) + \cdots. \tag{C.14}$$

We defined the function $\tilde{E}_1(A,U)$ as

$$\tilde{E}_1(A,U) = E_1(A,U) - \pi \ln\left(1 + 2\pi\chi\frac{\sqrt{G}U_2}{|A|^2}\right), \tag{C.15}$$

where $E_1(A,U)$ was defined in (2.65) for $s=1$. Note that $\tilde{E}_1(A,U)$ has a smooth limit for $A=0$. To see this one has to make use of

$$E_1(A,U) = -\pi \ln\left|\frac{\vartheta_1(A|U)}{\eta(U)}\right|^2 + 2\pi^2 U_2 a_4^2, \tag{C.16}$$

and then proceed as in [20] to show that

$$\tilde{E}_1(0,U) = -\frac{\pi}{T_2}\ln(8\pi^3\chi T_2 U_2|\eta(U)|^4)\,. \tag{C.17}$$

On the other hand, for $A \neq 0$, it has the expansion

$$\tilde{E}_1(A,U) = E_1(A,U) + \mathcal{O}\bigg(\frac{\chi\sqrt{G}U_2}{|A|^2}\bigg)\,, \tag{C.18}$$

i.e. there is no need for an IR cutoff χ which can be set to zero in the corresponding terms. Altogether, the building block correlators of (C.3)–(C.7), turn out to be

$$\begin{aligned}
\langle V^{(0,0)}_{ZZ} V^{(0,0)}_{\bar{Z}\bar{Z}}\rangle_\sigma &= (p_1\cdot p_2)\alpha'\frac{V_4}{(4\pi^2\alpha')^2}\frac{3c_\sigma^2\pi}{16T_2}\sum_k \mathrm{tr}\left[E_2(\mathbf{A}_\sigma,U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right],\\
\langle V^{(0)B}_{Z} V^{(0)C}_{\bar{Z}}\rangle_\sigma &= -(p_1\cdot p_2)\alpha'\frac{V_4}{(4\pi^2\alpha')^2}\frac{d_\sigma^2\pi}{8}\sum_k \mathrm{tr}\left[\lambda_{s[B]}\lambda^\dagger_{s[C]}\tilde{E}_1(\mathbf{A}_\sigma,U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right],\\
\langle V^{(0,0)}_{ZZ} V^{(0,0)}_{Z\bar{Z}}\rangle_\sigma &= (p_1\cdot p_2)\alpha'\frac{V_4}{(4\pi^2\alpha')^2}\frac{c_\sigma^2\pi U_2^2}{4T_2}\times\\
&\quad\times\sum_k \mathrm{tr}\left[\sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i\vec{\mathbf{A}}_\sigma\cdot\vec{n}}}{(n+mU)^3(n+m\bar{U})}\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right],\\
\langle V^{(0,0)}_{ZZ} V^{(0)B}_{\bar{Z}}\rangle_\sigma &= i(p_1\cdot p_2)\alpha'\frac{V_4}{(4\pi^2\alpha')^2}\frac{c_\sigma^3 d_\sigma\pi U_2^{3/2}}{8T_2^{1/2}}\times\\
&\quad\times\sum_k \mathrm{tr}\left[\lambda^\dagger_{s[B]}\sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i\vec{\mathbf{A}}_\sigma\cdot\vec{n}}}{|n+mU|^4}(n+m\bar{U})\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right],\\
\langle V^{(0)B}_{Z} V^{(0,0)}_{Z\bar{Z}}\rangle_\sigma &= i(p_1\cdot p_2)\alpha'\frac{V_4}{(4\pi^2\alpha')^2}\frac{c_\sigma^3 d_\sigma\pi U_2^{3/2}}{8T_2^{1/2}}\times\\
&\quad\times\sum_k \mathrm{tr}\left[\lambda_{s[B]}\sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i\vec{\mathbf{A}}_\sigma\cdot\vec{n}}}{|n+mU|^4}(n+m\bar{U})\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right].
\end{aligned} \tag{C.19}$$

In the 2-point functions for the open string moduli, the only contributions now come from

$$\sum_{i,j}\mathcal{A}^{(1)}_{ij} + \sum_{i,a}\left[\mathcal{A}^{(0)}_{ai}+\mathcal{A}^{(0)}_{ia}\right] + \sum_i \mathcal{M}^{(1)}_i\,. \tag{C.20}$$

To evaluate them we have to know the form of the Chan-Paton matrices. They are given by

$$\begin{aligned}
\mathcal{M}:\quad \lambda_{s[1]} &= \mathrm{diag}(\mathbf{1}_{N_{s[1]}},-\mathbf{1}_{N_{s[1]}})\oplus\mathbf{0}_{32-2N_{s[1]}}\,,\\
\mathcal{A}:\quad \lambda_{s[1]} &= \Big(\mathrm{diag}(\mathbf{1}_{N_{s[1]}},-\mathbf{1}_{N_{s[1]}})\oplus\mathbf{0}_{32-2N_{s[1]}}\Big)\otimes\mathbf{1}_{32}\,,\\
\lambda_{s[2]} &= \mathbf{1}_{32}\otimes\Big(-\mathrm{diag}(\mathbf{1}_{N_{s[2]}},-\mathbf{1}_{N_{s[2]}})\oplus\mathbf{0}_{32-2N_{s[2]}}\Big)\,,
\end{aligned} \tag{C.21}$$

where the extra minus in $\lambda_{s[2]}$ comes from the charge q_B at the string end point (compare the comment below equations (2.34)). These matrices then lead to

$$\sum_\sigma d_\sigma^2 \sum_{k=0,1} \mathrm{tr}\left[\lambda_{s[B]}\lambda_{s[C]}^\dagger \tilde{E}_1(\mathbf{A}_\sigma,U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right] =$$

$$\begin{aligned}
&= -4\sum_{i,j} N_i N_j \Big[(\delta_{is[B]}\delta_{is[C]} + \delta_{js[B]}\delta_{js[C]})[\tilde{E}_1(A_i - A_j,U) + \tilde{E}_1(-A_i + A_j,U) - \\
&\qquad\qquad - \tilde{E}_1(A_i + A_j,U) - \tilde{E}_1(-A_i - A_j,U)] - \\
&\qquad - (\delta_{is[B]}\delta_{js[C]} + \delta_{js[B]}\delta_{is[C]})[\tilde{E}_1(A_i - A_j,U) + \tilde{E}_1(-A_i + A_j,U) + \\
&\qquad\qquad + \tilde{E}_1(A_i + A_j,U) + \tilde{E}_1(-A_i - A_j,U)]\Big] + \\
&\quad +2\cdot 32\sum_i N_i \delta_{is[B]}\delta_{is[C]}[\tilde{E}_1(A_i,U) + \tilde{E}_1(-A_i,U)] - \\
&\quad -16\sum_i N_i \delta_{is[B]}\delta_{is[C]}[\tilde{E}_1(2A_i,U) + \tilde{E}_1(-2A_i,U)]\,.
\end{aligned} \tag{C.22}$$

For future reference, we introduce a closely related quantity, where both lambda matrices are inserted on the same stack of branes that we choose to be the ith stack, i.e. $s[B] = s[C] = i$,

$$\begin{aligned}
\mathcal{E}_1^{(i)}(A_l,U) &= \sum_\sigma d_\sigma^2 \sum_{k=0,1} \mathrm{tr}\left[\lambda_i\lambda_i^\dagger \tilde{E}_1(\mathbf{A}_\sigma,U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\right] \\
&= -8N_i\sum_l N_l[\tilde{E}_1(A_i - A_l,U) + \tilde{E}_1(-A_i + A_l,U) - \\
&\qquad\qquad - \tilde{E}_1(A_i + A_l,U) - \tilde{E}_1(-A_i - A_l,U)] + \\
&\quad +8N_i^2[2\tilde{E}_1(0,U) + \tilde{E}_1(2A_i,U) + \tilde{E}_1(-2A_i,U)] + \\
&\quad +64N_i[\tilde{E}_1(A_i,U) + \tilde{E}_1(-A_i,U)] - \\
&\quad -16\delta_{ij}N_i[\tilde{E}_1(2A_i,U) + \tilde{E}_1(-2A_i,U)]\,.
\end{aligned} \tag{C.23}$$

The UV divergences of the 95 annulus and the Möbius strip proportional to Λ^2 cancel against each other. All other terms involving the UV cutoff become proportional to $\sum_i N_i\partial_\mu A_i$ when inserted into the effective lagrangian, which is zero due to the anomaly constraint

$$\sum_i N_i A_i = 0\,, \tag{C.24}$$

that ensures a decoupling of the anomalous overall U(1) in U(16) [29].

We next evaluate the traces of the other expressions in (C.19)

$$\sum_\sigma c_\sigma^2 \sum_{k=0,1} \mathrm{tr}\Big[\sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i\vec{\mathbf{A}}_\sigma\cdot\vec{n}}}{(n+mU)^3(n+m\bar{U})}\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\Big] =$$

$$\begin{aligned}
&= -4\sum_{i,j} N_i N_j \sum_{\vec{n}=(n,m)^T}{}' \frac{1}{(n+mU)^3(n+m\bar{U})}\Big[e^{2\pi i(\vec{a}_i-\vec{a}_j)\cdot\vec{n}} + e^{2\pi i(-\vec{a}_i+\vec{a}_j)\cdot\vec{n}} - \\
&\qquad\qquad - e^{2\pi i(\vec{a}_i+\vec{a}_j)\cdot\vec{n}} - e^{2\pi i(-\vec{a}_i-\vec{a}_j)\cdot\vec{n}}\Big] +
\end{aligned}$$

$$
\begin{aligned}
&+2\cdot 32\sum_i N_i \sum_{\vec{n}=(n,m)^T}{}' \frac{1}{(n+mU)^3(n+m\bar{U})}\Big[e^{2\pi i\vec{a}_i\cdot\vec{n}}+e^{-2\pi i\vec{a}_i\cdot\vec{n}}\Big]-\\
&-4\sum_i N_i \sum_{\vec{n}=(n,m)^T}{}' \frac{1}{(n+mU)^3(n+m\bar{U})}\Big[e^{4\pi i\vec{a}_i\cdot\vec{n}}+e^{-4\pi i\vec{a}_i\cdot\vec{n}}\Big], \qquad \text{(C.25)}
\end{aligned}
$$

$$
\begin{aligned}
\sum_\sigma c_\sigma^3 d_\sigma \sum_{k=0,1} \operatorname{tr}\Big[\lambda_{s[B]} \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i\vec{\mathbf{A}}_\sigma\cdot\vec{n}}}{|n+mU|^4}(n+m\bar{U})\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k}\Big] =\\
= -4\sum_{i,j} N_iN_j \sum_{\vec{n}=(n,m)^T}{}' \frac{n+m\bar{U}}{|n+mU|^4}\Big(\delta_{is[B]}\Big[e^{2\pi i(\vec{a}_i-\vec{a}_j)\cdot\vec{n}}-e^{2\pi i(-\vec{a}_i+\vec{a}_j)\cdot\vec{n}}-\\
-e^{2\pi i(\vec{a}_i+\vec{a}_j)\cdot\vec{n}}+e^{2\pi i(-\vec{a}_i-\vec{a}_j)\cdot\vec{n}}\Big]-\\
-\delta_{js[B]}\Big[e^{2\pi i(\vec{a}_i-\vec{a}_j)\cdot\vec{n}}-e^{2\pi i(-\vec{a}_i+\vec{a}_j)\cdot\vec{n}}+\\
+e^{2\pi i(\vec{a}_i+\vec{a}_j)\cdot\vec{n}}-e^{2\pi i(-\vec{a}_i-\vec{a}_j)\cdot\vec{n}}\Big]\Big)+\\
+2\cdot 32\sum_i N_i\delta_{is[B]} \sum_{\vec{n}=(n,m)^T}{}' \frac{n+m\bar{U}}{|n+mU|^4}\Big[e^{2\pi i\vec{a}_i\cdot\vec{n}}-e^{-2\pi i\vec{a}_i\cdot\vec{n}}\Big]-\\
-8\sum_i N_i\delta_{is[B]} \sum_{\vec{n}=(n,m)^T}{}' \frac{n+m\bar{U}}{|n+mU|^4}\Big[e^{4\pi i\vec{a}_i\cdot\vec{n}}-e^{-4\pi i\vec{a}_i\cdot\vec{n}}\Big]. \qquad \text{(C.26)}
\end{aligned}
$$

Also both of these traces only get contributions from the 99- and 95-annulus and the 9-brane Möbius strip. Moreover, the trace in (C.26) with $\lambda_{s[B]}$ replaced by $\lambda^\dagger_{s[B]}$ gives the same result.

Putting everything together, we finally get

$$
\begin{aligned}
\langle V_U V_{\bar{U}}\rangle = (p_1\cdot p_2)\alpha' g_c^2\alpha'^{-4}\frac{V_4}{(4\pi^2\alpha')^2}e^{-\Phi}\times\\
\times\Big[-\frac{3i}{4\sqrt{2}(U-\bar{U})^2(S_0'-\bar{S}_0')}\mathcal{E}_2(A_k,U)+\frac{\pi}{4\sqrt{2}(U-\bar{U})(S_0'-\bar{S}_0')}\times\\
\times\sum_{\vec{n}=(n,m)^T}{}' \frac{2n+m(U+\bar{U})}{|n+mU|^4}\times\\
\times\Bigg(-4\sum_{i,j}N_iN_j\Big[(A_i-\bar{A}_i-(A_j-\bar{A}_j))(e^{2\pi i(\vec{a}_i-\vec{a}_j)\cdot\vec{n}}-e^{2\pi i(-\vec{a}_i+\vec{a}_j)\cdot\vec{n}})-\\
-(A_i-\bar{A}_i+(A_j-\bar{A}_j))(e^{2\pi i(\vec{a}_i+\vec{a}_j)\cdot\vec{n}}-e^{2\pi i(-\vec{a}_i-\vec{a}_j)\cdot\vec{n}})\Big]+\\
+64\sum_i N_i(A_i-\bar{A}_i)(e^{2\pi i\vec{a}_i\cdot\vec{n}}-e^{-2\pi i\vec{a}_i\cdot\vec{n}})-\\
-8\sum_i N_i(A_i-\bar{A}_i)(e^{4\pi i\vec{a}_i\cdot\vec{n}}-e^{-4\pi i\vec{a}_i\cdot\vec{n}})\Bigg)+\frac{\pi^2}{\sqrt{2}(U-\bar{U})^3(S_0'-\bar{S}_0')}\times\\
\times\Bigg(-4\sum_{i,j}N_iN_j\Big[(A_i-\bar{A}_i-(A_j-\bar{A}_j))^2\times
\end{aligned}
$$

$$
\begin{aligned}
&\times (\tilde{E}_1(A_i - A_j, U) + \tilde{E}_1(-A_i + A_j, U)) - \\
&- (A_i - \bar{A}_i + (A_j - \bar{A}_j))^2 \times \\
&\times (\tilde{E}_1(A_i + A_j, U) + \tilde{E}_1(-A_i - A_j, U))\Big] + \\
&+ 64 \sum_i N_i (A_i - \bar{A}_i)^2 (\tilde{E}_1(A_i, U) + \tilde{E}_1(-A_i, U)) - \\
&- 16 \sum_i N_i (A_i - \bar{A}_i)^2 (\tilde{E}_1(2A_i, U) + \tilde{E}_1(-2A_i, U))\Bigg)\Bigg], \qquad \text{(C.27)}
\end{aligned}
$$

$$
\begin{aligned}
\langle V_{A_i} V_{\bar{A}_j} \rangle = {} & (p_1 \cdot p_2) \alpha' g_o^2 \alpha'^{-4} \frac{V_4}{(4\pi^2\alpha')^2} e^{-\Phi} \frac{1}{\sqrt{2}(U - \bar{U})(S_0' - \bar{S}_0')} \times \\
& \times \Big[- 8\delta_{ij} N_i \sum_l N_l \Big[\tilde{E}_1(A_i - A_l, U) + \tilde{E}_1(-A_i + A_l, U) - \\
& \qquad - \tilde{E}_1(A_i + A_l, U) - \tilde{E}_1(-A_i - A_l, U)\Big] + \\
& + 8 N_i N_j \Big[\tilde{E}_1(A_i - A_j, U) + \tilde{E}_1(-A_i + A_j, U) + \\
& \qquad + \tilde{E}_1(A_i + A_j, U) + \tilde{E}_1(-A_i - A_j, U)\Big] + \\
& + 64 \delta_{ij} N_i \Big[\tilde{E}_1(A_i, U) + \tilde{E}_1(-A_i, U)\Big] - \\
& - 16 \delta_{ij} N_i \Big[\tilde{E}_1(2A_i, U) + \tilde{E}_1(-2A_i, U)\Big]\Big], \qquad \text{(C.28)}
\end{aligned}
$$

$$
\begin{aligned}
\langle V_U V_{S_2'} \rangle = {} & (p_1 \cdot p_2) \alpha' g_c^2 \alpha'^{-4} \frac{V_4}{(4\pi^2\alpha')^2} e^{-\Phi} \times \\
& \times \Bigg[\frac{i\pi}{8\sqrt{2}(S_0' - \bar{S}_0')^2} \sum_{\vec{n}=(n,m)^T}{}' \frac{n + m\bar{U}}{|n + mU|^4} \times \\
& \times \Bigg(- 4 \sum_{i,j} N_i N_j \Big[(A_i - \bar{A}_i - (A_j - \bar{A}_j))(e^{2\pi i(\vec{a}_i - \vec{a}_j)\cdot\vec{n}} - e^{2\pi i(-\vec{a}_i + \vec{a}_j)\cdot\vec{n}}) - \\
& \qquad - (A_i - \bar{A}_i + (A_j - \bar{A}_j))(e^{2\pi i(\vec{a}_i + \vec{a}_j)\cdot\vec{n}} - e^{2\pi i(-\vec{a}_i - \vec{a}_j)\cdot\vec{n}})\Big] + \\
& + 64 \sum_i N_i (A_i - \bar{A}_i)(e^{2\pi i \vec{a}_i\cdot\vec{n}} - e^{-2\pi i \vec{a}_i\cdot\vec{n}}) - \\
& - 8 \sum_i N_i (A_i - \bar{A}_i)(e^{4\pi i \vec{a}_i\cdot\vec{n}} - e^{-4\pi i \vec{a}_i\cdot\vec{n}})\Bigg) - \frac{(U - \bar{U})}{8\sqrt{2}(S_0' - \bar{S}_0')^2} \times \\
& \times \Bigg(- 4 \sum_{i,j} N_i N_j \sum_{\vec{n}=(n,m)^T}{}' \frac{1}{(n + mU)^3 (n + m\bar{U})} \times \\
& \times \Big[e^{2\pi i(\vec{a}_i - \vec{a}_j)\cdot\vec{n}} + e^{2\pi i(-\vec{a}_i + \vec{a}_j)\cdot\vec{n}} - e^{2\pi i(\vec{a}_i + \vec{a}_j)\cdot\vec{n}} - e^{2\pi i(-\vec{a}_i - \vec{a}_j)\cdot\vec{n}}\Big] + \\
& + 64 \sum_i N_i \sum_{\vec{n}=(n,m)^T}{}' \frac{1}{(n + mU)^3 (n + m\bar{U})} [e^{2\pi i \vec{a}_i\cdot\vec{n}} + e^{-2\pi i \vec{a}_i\cdot\vec{n}}] -
\end{aligned}
$$

$$-4\sum_i N_i \sum_{\vec{n}=(n,m)^T}{}' \frac{1}{(n+mU)^3(n+m\bar{U})}[e^{4\pi i\vec{a}_i\cdot\vec{n}}+e^{-4\pi i\vec{a}_i\cdot\vec{n}}]\Bigg)\Bigg], \quad \text{(C.29)}$$

$$\begin{aligned}
\langle V_U V_{\bar{A}_i}\rangle &= (p_1\cdot p_2)\alpha' g_c g_o \alpha'^{-4}\frac{V_4}{(4\pi^2\alpha')^2}e^{-\Phi}\times \\
&\times\Bigg[-\frac{1}{4\sqrt{2}(S_0'-\bar{S}_0')}\sum_{\vec{n}=(n,m)^T}{}'\frac{n+m\bar{U}}{|n+mU|^4}\times \\
&\times\Bigg(-8\sum_l N_i N_l[e^{2\pi i(\vec{a}_i-\vec{a}_l)\cdot\vec{n}}-e^{2\pi i(-\vec{a}_i+\vec{a}_l)\cdot\vec{n}}-e^{2\pi i(\vec{a}_i+\vec{a}_l)\cdot\vec{n}}+e^{2\pi i(-\vec{a}_i-\vec{a}_l)\cdot\vec{n}}]+ \\
&\quad+64N_i[e^{2\pi i\vec{a}_i\cdot\vec{n}}-e^{-2\pi i\vec{a}_i\cdot\vec{n}}]-8N_i[e^{4\pi i\vec{a}_i\cdot\vec{n}}-e^{-4\pi i\vec{a}_i\cdot\vec{n}}]\Bigg)- \\
&-\frac{\pi}{\sqrt{2}(U-\bar{U})^2(S_0'-\bar{S}_0')}\times \\
&\times\Bigg(-8N_i\sum_l N_l\Big[(A_i-\bar{A}_i-(A_l-\bar{A}_l))(\tilde{E}_1(A_i-A_l,U)+ \\
&\qquad+\tilde{E}_1(-A_i+A_l,U))- \\
&\qquad-(A_i-\bar{A}_i+(A_l-\bar{A}_l))(\tilde{E}_1(A_i+A_l,U)+ \\
&\qquad+\tilde{E}_1(-A_i-A_l,U))\Big]+ \\
&\quad+64N_i\Big[\tilde{E}_1(A_i,U)+\tilde{E}_1(-A_i,U)\Big]- \\
&\quad-16N_i\Big[\tilde{E}_1(2A_i,U)+\tilde{E}_1(-2A_i,U)\Big]\Bigg)\Bigg], \qquad \text{(C.30)}
\end{aligned}$$

$$\begin{aligned}
\langle V_{A_i} V_{S_2'}\rangle &= (p_1\cdot p_2)\alpha' g_c g_o \alpha'^{-4}\frac{V_4}{(4\pi^2\alpha')^2}e^{-\Phi}\times \\
&\times\Bigg[-\frac{i(U-\bar{U})}{8\sqrt{2}(S_0'-\bar{S}_0')^2}\sum_{\vec{n}=(n,m)^T}{}'\frac{n+m\bar{U}}{|n+mU|^4}\times \\
&\times\Bigg(-8\sum_l N_i N_l\Big[e^{2\pi i(\vec{a}_i-\vec{a}_l)\cdot\vec{n}}-e^{2\pi i(-\vec{a}_i+\vec{a}_l)\cdot\vec{n}}-e^{2\pi i(\vec{a}_i+\vec{a}_l)\cdot\vec{n}}+ \\
&\qquad+e^{2\pi i(-\vec{a}_i-\vec{a}_l)\cdot\vec{n}}\Big]+64N_i\Big[e^{2\pi i\vec{a}_i\cdot\vec{n}}-e^{-2\pi i\vec{a}_i\cdot\vec{n}}\Big]- \\
&\quad-8N_i\Big[e^{4\pi i\vec{a}_i\cdot\vec{n}}-e^{-4\pi i\vec{a}_i\cdot\vec{n}}\Big]\Bigg)\Bigg]. \qquad \text{(C.31)}
\end{aligned}$$

To see that this is consistent with (C.1), note that

$$\begin{aligned}
\partial_U\partial_{\bar{U}}E_2(A,U) &= -\frac{2}{(U-\bar{U})^2}E_2(A,U)-\frac{2\pi^2 i(A-\bar{A})^2}{(U-\bar{U})^3}E_1(A,U)- \\
&-\frac{\pi i}{2}\frac{A-\bar{A}}{U-\bar{U}}\sum_{\vec{n}=(n,m)^T}{}'\frac{e^{2\pi i\vec{a}\cdot\vec{n}}}{|n+mU|^4}(2n+m(U+\bar{U})),
\end{aligned}$$

$$
\begin{aligned}
\partial_A \partial_{\bar{A}} E_2(A,U) &= \frac{-2\pi^2 i}{U-\bar{U}} E_1(A,U)\,, \\
\partial_U E_2(A,U) &= \frac{\pi i (A-\bar{A})}{2} \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i \vec{a}\cdot\vec{n}}}{(n+m\bar{U})(n+mU)^2} - \\
&\quad - \frac{U-\bar{U}}{2} \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i \vec{a}\cdot\vec{n}}}{(n+m\bar{U})(n+mU)^3}\,, \\
\partial_U \partial_{\bar{A}} E_2(A,U) &= \frac{2\pi^2 i (A-\bar{A})}{(U-\bar{U})^2} E_1(A,U) + \frac{\pi i}{2} \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i \vec{a}\cdot\vec{n}}}{(n+m\bar{U})(n+mU)^2}\,, \\
\partial_A E_2(A,U) &= -\frac{\pi i (U-\bar{U})}{2} \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i \vec{a}\cdot\vec{n}}}{(n+m\bar{U})(n+mU)^2}\,.
\end{aligned}
\tag{C.32}
$$

To compare this (in combination with (C.1)) with (C.27)–(C.31) we have to perform a Weyl rescaling in the latter. For most cases this just amounts to a rescaling with the factor given in (2.70). However, as for the case of S' discussed in the main text, also for the kinetic term of U the correction to the Einstein Hilbert term (2.71), calculated in [10], has to be taken into account. Again the reason is the presence of the kinetic term for U already at sphere level, cf. (2.16). To make this more precise let us have a look at the relevant terms in the effective action (using $\mathcal{V} = \mathcal{V}_{\mathrm{K3}}\sqrt{G}$)

$$
\begin{aligned}
&\frac{1}{2}\left[e^{-2\Phi}\mathcal{V} + \frac{\tilde{c}}{\sqrt{G}}\mathcal{E}_2(A_i,U)\right] R + \left[\frac{e^{-2\Phi}\mathcal{V}}{(U-\bar{U})^2} + \frac{(\tilde{c}+\tilde{c}_0)\mathcal{E}_2(A_i,U)}{\sqrt{G}(U-\bar{U})^2} + \cdots\right]\partial_\mu U \partial^\mu \bar{U} \\
\overset{\text{Weyl}}{\longrightarrow}\;& \frac{1}{2}R + \left[\frac{1}{(U-\bar{U})^2} + \frac{\tilde{c}_0 \mathcal{E}_2(A_i,U) e^{2\Phi}}{(U-\bar{U})^2\sqrt{G}\mathcal{V}} + \cdots\right]\partial_\mu U \partial^\mu \bar{U}\,,
\end{aligned}
\tag{C.33}
$$

where we only displayed the part of the one-loop correction to the kinetic term of U that is proportional to $\mathcal{E}_2(A_i,U)$ and omitted terms of order $\mathcal{O}(e^{3\Phi})$. Furthermore, the coefficient $\tilde{c}+\tilde{c}_0$ represents a split into the contribution to the correlator $\langle V^{(0,0)}_{ZZ} V^{(0,0)}_{\bar{Z}\bar{Z}}\rangle_\sigma$ coming from fluctuations ($\tilde{c}$) and zero modes ($\tilde{c}_0$), cf. (2.61). The correction to the Einstein-Hilbert term only comes from fluctuations (there are no zero modes along the non-compact directions). This means that after the Weyl rescaling, only zero mode contributions survive in the kinetic term of U (this is analogous to the case of S', discussed in the main text, cf. footnote 18). This gives exactly the right relative factor compared to the other contributions in (C.27), i.e. those that are not proportional to $\mathcal{E}_2(A_i,U)$, in order to be consistent with (C.32) (to be more precise, the factor $-3i/(4\sqrt{2})$ in the first line of (C.27) becomes $-i/\sqrt{2}$).

In order to read off the kinetic terms from the amplitudes (C.27)–(C.31) we have to make the replacements

$$
V_4 \;\to\; d^4x\sqrt{-g} \tag{C.34}
$$

and

$$
\begin{aligned}
(p_1\cdot p_2) g_c^2 \pi^2 \alpha'^{-4} &\to \partial_\mu U \partial^\mu \bar{U}\,, \qquad \text{in (C.27)}\,, \\
(p_1\cdot p_2) g_o^2 \alpha'^{-4} &\to \partial_\mu A_i \partial^\mu \bar{A}_j\,, \qquad \text{in (C.28)}\,,
\end{aligned}
$$

$$\begin{aligned}
\frac{1}{2}(p_1\cdot p_2)g_c^2\pi^2\alpha'^{-4} &\to \partial_\mu U\partial^\mu S_2'\,, \qquad \text{in (C.29)}\,,\\
(p_1\cdot p_2)g_cg_o\pi\alpha'^{-4} &\to \partial_\mu U\partial^\mu \bar{A}_i\,, \qquad \text{in (C.30)}\,,\\
\frac{1}{2}(p_1\cdot p_2)g_cg_o\pi\alpha'^{-4} &\to \partial_\mu A_i\partial^\mu S_2'\,, \qquad \text{in (C.31)}\,.
\end{aligned} \tag{C.35}$$

Again we neglected overall numerical factors. The additional factor of $\frac{1}{2}$ on the left hand sides in those cases that involve an S_2' should ultimately be checked by comparing with the corresponding normalization at tree level. In order to compare with the metrics (C.1), one finally has to take into account that

$$G_{U\bar{S}'}\partial_\mu U\partial^\mu\bar{S}' = -iG_{U\bar{S}'}\partial_\mu U\partial^\mu S_2' + G_{U\bar{S}'}\partial_\mu U\partial^\mu S_1'\,. \tag{C.36}$$

This means that we still have to multiply the results (C.29) and (C.31), i.e. the correlators involving one S_2', by i in order to read off the actual Kähler metric. Taking all this into account one verifies that our results (C.27)–(C.31) are reproduced by the Kähler potential (2.77) (up to a common numerical constant that we did not determine).

D. Fixing the constant c

In section 2.5 we have determined the correction to the Kähler potential up to a numerical constant c which we left free in the final result (2.77). It would be nice to fix it by direct comparison of the tree and one-loop contributions to the Kähler metric. However, here we use an indirect method and make use of the relation (2.15) that relates the metric to the gauge couplings of D9-brane gauge groups, which reads in our case

$$\begin{aligned}
K_{A_i\bar{A}_i}\Big|_{A_i=0} &= -\frac{1}{4\pi}\frac{(S-\bar{S})-4\pi c(U-\bar{U})\partial_{A_i}\partial_{\bar{A}_i}\mathcal{E}_2(A_j,U)}{(S-\bar{S})(S'-\bar{S}')(U-\bar{U})}\Big|_{A_i=0}+\mathcal{O}(e^{3\Phi})\,,\\
e^K\,\mathrm{Re}(f_{\mathrm{D}9_i})\Big|_{A_i=0} &= \frac{1}{2i}\frac{(S-\bar{S})+2i\mathrm{Re}(f^{(1)}_{\mathrm{D}9_i})}{(S-\bar{S})(S'-\bar{S}')(U-\bar{U})}\Big|_{A_i=0}+\mathcal{O}(e^{3\Phi})\,.
\end{aligned} \tag{D.1}$$

This equality, that we expanded to order $e^{2\Phi}$, is required to hold exactly and thereby fixes the constant c. At order $e^{2\Phi}$ we have

$$2\pi ic\,(U-\bar{U})\partial_{A_i}\partial_{\bar{A}_i}\mathcal{E}_2(A_j,U)\Big|_{A_i=0} = \mathrm{Re}(f^{(1)}_{\mathrm{D}9_i})\Big|_{A_i=0}\,. \tag{D.2}$$

Using the relation (C.32) one can rewrite the left-hand-side as a function of $E_1(A,U)$. Thus, (D.2) becomes

$$4\pi^3c\,\mathcal{E}_1^{(i)}(0,U) = \mathrm{Re}(f^{(1)}_{\mathrm{D}9_i})\Big|_{A_i=0}\,, \tag{D.3}$$

$\mathcal{E}_1^{(i)}(A_l,U)$ defined in (C.23). Now we look to our results of [20, 56] where we calculated the one-loop corrections to the gauge couplings, and use them for comparison. One needs to be careful about the fact that we are dealing with the $\mathrm{U}(1)_i$ subgroups of the $\mathrm{U}(N_i)$ gauge group factors. Their couplings were computed in section 3.3.2 of [20]. To compare to the form of $\mathcal{E}_1^{(i)}(A_l,U)$ in (C.23) it is easiest to consider the set of equations (36) in [20] whose sum produces the one-loop corrections to the gauge kinetic functions up to a factor of 2 (that can be inferred from formula (13) of [20]). One only has to replace the theta functions by

$$\int_0^{\Lambda^2} dl\, \vartheta\left[\begin{smallmatrix}\vec{0}\\ \vec{0}\end{smallmatrix}\right](\vec{a}, ie_\sigma lG)e^{-\pi\chi/(e_\sigma l)} = \frac{1}{e_\sigma}\left[e_\sigma\Lambda^2 + \frac{1}{\pi T_2}\tilde{E}_1(A,U) + \cdots\right] \tag{D.4}$$

in that formula.[33] With this in mind, one finds that

$$\mathrm{Re}(f^{(1)}_{\mathrm{D}9_i}) = \frac{1}{32\pi^3}\mathcal{E}^{(i)}_1(A_l, U)\,. \tag{D.5}$$

This factor can be understood as follows. There is one overall factor 2 that we mentioned above. For the annulus $\mathcal{A}_{95}$ the factor follows immediately from the factor in (36) of [20] and (D.4) (with $e_{(ia)} = 1$), whereas for $\mathcal{A}_{99}$ and $\mathcal{M}_9$, the factor consists of

$$2\frac{e_\sigma^2}{16\pi^2}\frac{1}{4e_\sigma}\frac{1}{e_\sigma\pi}\,, \tag{D.6}$$

where a factor $-e_\sigma^2/(16\pi^2)$ comes from (36) of [20], the $1/(4e_\sigma)$ from cancelling the factors $d_\sigma^2\,\mathcal{Q}_{\sigma,k}(= -4e_\sigma)$ in (C.23) and the $1/(e_\sigma\pi)$ from (D.4). Plugging (D.5) back into (D.1), we finally find

$$c = \frac{1}{128\pi^6}\,. \tag{D.7}$$

An important consistency check of the direct calculation of [20] was that the one-loop diagrams provide the terms in the gauge couplings of D5-branes involving the Wilson line moduli A_i in the definition of the Kähler variable S' compared to S'_0. This fixes the relative coefficient of the tree-level and one-loop gauge couplings uniquely with no room for other factors.

E. A 4-point check

In this appendix we would like to check the validity of relaxation of momentum conservation, used throughout the main text. We do this by calculating a specific 4-point function (for which relaxation is not needed) and comparing the result with the one derived from the 2-point function using the relaxation of momentum conservation. The simplest amplitude to consider, that does not require a generalization of (2.62), is the 4-point function of three open string scalars and one $\bar{S}'$. More precisely, we will check only the 99 annulus $\mathcal{A}_{ij}$ with two open string vertices inserted on the left and one on the right side. Thus we are concentrating on[34]

$$\langle V_{S'_2}V_{A_j}V_{A_i}V_{\bar{A}_i}\rangle_{\mathcal{A}_{ij}} \sim \frac{ig_o^3 g_c(\alpha')^{-8}}{(S'_0-\bar{S}'_0)(U-\bar{U})^{3/2}(T-\bar{T})^{3/2}}\langle V^{(0,0)}_{Z\bar{Z}}V^{(0)j}_{Z}V^{(0)i}_{Z}V^{(0)i}_{\bar{Z}}\rangle_{\mathcal{A}_{ij}}\,. \tag{E.1}$$

[33] Note that we now use slightly different conventions compared to [20] for the modular transformation and the UV cutoff. The terms proportional to Λ^2 drop out when summed over all diagrams.

[34] The notation for the open string vertex operators deviates slightly from (2.32) in an obvious way, because we do not need the general notation here, when we are only considering the $\mathcal{A}_{ij}$ diagram.

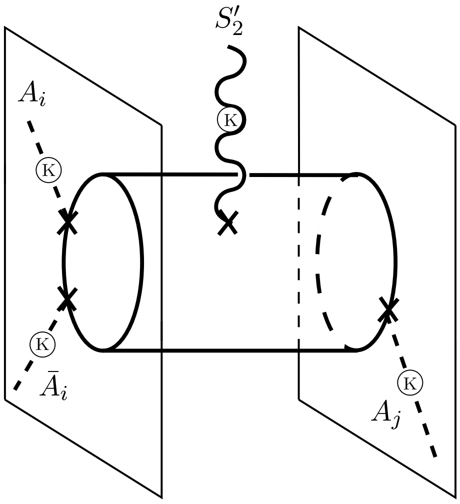

Figure 4: The 4-point function $\langle V_{S_2'} V_{A_j} V_{A_i} V_{\bar{A}_i}\rangle$. Unlike in fig 1, the A_i vertex operators are drawn inserted on the boundary of the cylinder to show which D-brane they belong to. Since the vertex operators in (2.34) contain both open and closed string vertex operators, it is immaterial whether we draw them with dashed or wiggly lines.

We will only check the moduli dependence and do not keep track of the overall factor, except for factors of π and i. We will first consider the contribution from the $k = 1$ sector and calculate the term proportional to $p_3 \cdot p_4$. For the 4-point function this does not vanish due to momentum conservation. We will do the calculation by using the ψ-dependent terms of the last two vertex operators, which give the desired power in momenta, and then setting the momenta to zero in the other terms. We will see that this exactly reproduces the moduli dependence that one would get from taking the derivative of the $A_i \bar{A}_i$ metric with respect to A_j and $\bar{S}'$. Afterwards we will argue that there are no additional contributions to the term proportional to $p_3 \cdot p_4$, neither from the $k = 0$ nor from the $k = 1$ sector.

The aforementioned contribution from the $k = 1$ sector, i.e. the one derived from taking the ψ-dependent terms of the last two vertex operators and setting the momenta to zero elsewhere, is given by

$$
\begin{aligned}
\langle V^{(0,0)}_{Z\bar{Z}} V^{(0)j}_{Z} V^{(0)i}_{Z} V^{(0)i}_{\bar{Z}}\rangle_{\mathcal{A}_{ij}} \sim{}& (p_3 \cdot p_4)\alpha' \frac{V_4}{(4\pi^2\alpha')^2} T_2 \int_0^\infty \frac{dt}{t^4} \int_{\mathcal{F}_{(ij)}} d^2\nu_1 \int_{\partial \mathcal{A}_{ij}} d\tilde{\nu}_2 d\tilde{\nu}_3 d\tilde{\nu}_4 \times \\
&\times \sum_{\vec{n}=(n,m)^T} \Bigg[\mathrm{tr}\Bigg(\lambda_j \lambda_i \lambda_i^\dagger e^{-\pi \vec{n}^T G \vec{n} t^{-1}} e^{2\pi i \vec{\mathbf{A}}_{ij}\cdot \vec{n}} \gamma_{(ij),1} \times \\
&\qquad \times \sum_{\substack{\alpha\beta \\ \text{even}}} \frac{\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(0,\tau)}{\eta^3(\tau)} \mathcal{Z}^{\text{int}}_{\sigma,k}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \times \\
&\qquad \times \langle \psi(\nu_3)\psi(\nu_4)\rangle^{\alpha,\beta} \langle \Psi(\nu_3)\bar{\Psi}(\nu_4)\rangle^{\alpha,\beta} \Bigg) \times \\
&\quad \times i\alpha'^{-3/2} \Big[\langle \partial \bar{Z}(\nu_1) \bar{\partial} Z(\bar{\nu}_1) \dot{Z}(\nu_2)\rangle +
\end{aligned}
$$

$$
\begin{aligned}
&+ \langle \partial Z(\nu_1) \bar\partial \bar Z(\bar\nu_1) \dot Z(\nu_2)\rangle + \\
&+ \langle \partial \bar Z(\nu_1) \dot Z(\nu_2)\rangle \langle \bar\partial Z(\bar\nu_1)\rangle + \\
&+ \langle \bar\partial \bar Z(\bar\nu_1) \dot Z(\nu_2)\rangle \langle \partial Z(\nu_1)\rangle \Big] \Bigg] . \qquad \text{(E.2)}
\end{aligned}
$$

The bosonic correlators with an odd number of fields in the last two lines of (E.2) only get contributions from the zero modes (2.49). As in the case of the 2-point functions discussed before, the sum over spin structures only gives a factor of -4, cf. (2.58). Thus the integral over $\tilde\nu_3$ and $\tilde\nu_4$ is easily performed and gives a factor of $\pi^2 t^2$. Let us next calculate the bosonic correlators. They are given by (here we have to keep track of the exact factors in order to obtain the correct relative factors between the different contributions of the last and the penultimate line in (E.2))

$$
\begin{aligned}
\int_{\mathcal{F}_{(ij)}} d^2\nu_1 \int_{\partial\mathcal{A}_{ij}} d\tilde\nu_2 \langle \partial \bar Z(\nu_1) \bar\partial Z(\bar\nu_1) \dot Z(\nu_2)\rangle &= \int_{\mathcal{F}_{(ij)}} d^2\nu_1 \int_{\partial\mathcal{A}_{ij}} d\tilde\nu_2 \langle \partial Z(\nu_1) \bar\partial \bar Z(\bar\nu_1) \dot Z(\nu_2)\rangle \\
&= 2\pi^3 \alpha'^{3/2} \left(\frac{T_2}{2U_2}\right)^{3/2} \frac{|n+mU|^2 (n+m\bar U)}{t} \qquad \text{(E.3)}
\end{aligned}
$$

and

$$
\begin{aligned}
&\int_{\mathcal{F}_{(ij)}} d^2\nu_1 \int_{\partial\mathcal{A}_{ij}} d\tilde\nu_2 \Big[\langle \partial \bar Z(\nu_1) \dot Z(\nu_2)\rangle \langle \bar\partial Z(\bar\nu_1)\rangle + \langle \bar\partial \bar Z(\bar\nu_1) \dot Z(\nu_2)\rangle \langle \partial Z(\nu_1)\rangle \Big] = \\
&\quad = i\alpha'^{1/2} \left(\frac{T_2}{2U_2}\right)^{1/2} \frac{n+m\bar U}{t} \int_{\mathcal{F}_{(ij)}} d^2\nu_1 \int_{\partial\mathcal{A}_{ij}} d\tilde\nu_2 \Big[\langle \partial \bar Z(\nu_1) \dot Z(\nu_2)\rangle - \langle \bar\partial \bar Z(\bar\nu_1) \dot Z(\nu_2)\rangle \Big] \\
&\quad = -\pi^2 \alpha'^{3/2} \left(\frac{T_2}{2U_2}\right)^{1/2} (n+m\bar U) + 4\pi^3 \alpha'^{3/2} \left(\frac{T_2}{2U_2}\right)^{3/2} \frac{|n+mU|^2 (n+m\bar U)}{t} , \qquad \text{(E.4)}
\end{aligned}
$$

where there are two contributions to the 2-point correlators of Z, coming from fluctuations and zero modes, respectively. Plugging this into (E.2) and performing the t-integration in a similar way as in (2.64), we finally end up with[35]

$$
\begin{aligned}
\langle V^{(0,0)}_{Z\bar Z} V^{(0)j}_{Z} V^{(0)i}_{Z} V^{(0)i}_{\bar Z} \rangle_{\mathcal{A}_{ij}} \sim\ & i\pi^3 \frac{V_4}{(4\pi^2\alpha')^2} \sqrt{U_2 T_2} (p_3 \cdot p_4) \alpha' \times \\
& \times \operatorname{tr} \left(\lambda_j \lambda_i \lambda_i^\dagger \gamma_{(ij),1} \sum_{\vec n = (n,m)^T}{}' \frac{e^{2\pi i \vec{\mathbf{A}}_{(ij)} \cdot \vec n}}{n+mU} \right) . \qquad \text{(E.5)}
\end{aligned}
$$

The trace can be evaluated using the matrices of table 1 and formulas (2.51) and (C.21). If we also take into account the factor from the Weyl rescaling (2.70), we end up with

$$
\begin{aligned}
\langle V_{S_2'} V_{A_i} V_{A_i} V_{\bar A_i} \rangle_{\mathcal{A}_{ij}} \xrightarrow{\text{Weyl}}\ & (p_3 \cdot p_4)\alpha' \frac{V_4}{(4\pi^2\alpha')^2} \frac{\pi g_o^3 g_c (\alpha')^{-8}}{(S-\bar S)(S_0' - \bar S_0')^2 (U - \bar U)} N_i N_j \times \\
& \times \sum_{\vec n = (n,m)^T}{}' \left[\frac{1}{n+mU} \Big[e^{2\pi i (\vec a_i - \vec a_j)\cdot \vec n} - e^{2\pi i (-\vec a_i + \vec a_j)\cdot \vec n} + \right.
\end{aligned}
$$

[35]Note that there is no need for a regularization in this case because there is no contribution from a term with $m = n = 0$.

$$+ e^{2\pi i(\vec{a}_i+\vec{a}_j)\cdot\vec{n}} - e^{2\pi i(-\vec{a}_i-\vec{a}_j)\cdot\vec{n}}\Bigg]\Bigg] . \quad \text{(E.6)}$$

Using

$$\partial_A\partial_A\partial_{\bar{A}}E_2(A,U) = \frac{-2\pi^3 i}{U-\bar{U}} \sum_{\vec{n}=(n,m)^T}{}' \frac{e^{2\pi i\vec{a}\cdot\vec{n}}}{n+mU}\,, \quad \text{(E.7)}$$

and substitutions similar to (C.34) and (C.35), i.e.

$$i(p_3\cdot p_4)g_o^3\pi g_c\alpha'^{-8} \;\to\; \delta S_2'\delta A_j\partial_\mu A_i\partial^\mu \bar{A}_i\,, \quad \text{(E.8)}$$

it is straightforward to see that the moduli dependence of (E.6) exactly reproduces the one of the second derivative of the contribution from $\mathcal{A}_{ij}$ to the $A_i\bar{A}_i$ metric (given in the second line of (C.1)) with respect to A_j and $\bar{S}'$. The necessary extra factor of i in (E.8) is as discussed below (C.36).

Next, we would like to argue that there are no further contributions to the 4-point function coming from either the $k=0$ or $k=1$ sector. Let us begin with the $k=0$ sector. In order to get a non-vanishing result from the spin structure summation, one has to contract eight fermionic fields in (E.1). This already leads to a fourth power of momenta, e.g. to a term proportional to s^2, where $s\equiv -(p_1+p_2)^2 = -(p_3+p_4)^2$. In order to get a contribution to the kinetic term of the A_i, the integration over t would have to give a pole in s. This would require the exchange of a massless particle in the closed string tree channel. However, the massless particles reside in the sector with $m=n=0$. On the other hand, contracting eight of the fermionic world-sheet fields of the vertex operators in (E.1) would leave a bosonic correlator of the form $\langle\partial Z\rangle$ or $\langle\bar\partial Z\rangle$, which only gets contributions from the zero modes (2.49). Thus the amplitude trivially vanishes for $m=n=0$ and there is no contribution proportional to $p_3\cdot p_4$ from the $k=0$ sector.

A similar argument can be put forward for the $k=1$ sector. Again, an additional contribution could only come from contracting eight of the fermionic world-sheet fields of the vertex operators in (E.1) if there were a pole from the t-integration. Such a pole can be excluded in exactly the same way as for the $k=0$ sector. Alternatively, for the $k=1$ sector one could also argue that the exchanged massless particle would reside in a hypermultiplet from the twisted sector. Thus the exchange would require a coupling between hypermultiplets and the vector multiplets, whose scalars are external states in the 4-point function (E.1). This is, however, forbidden by supersymmetry. Either way, we conclude that there are no additional contributions to the 4-point function (E.1) proportional to $p_3\cdot p_4$ and (E.6) is the final result.

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